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DESIGN CRITERIA FOR A DEEP TOWED SOURCE AND MULTI-CHANNEL ARRAY--ETC(U)

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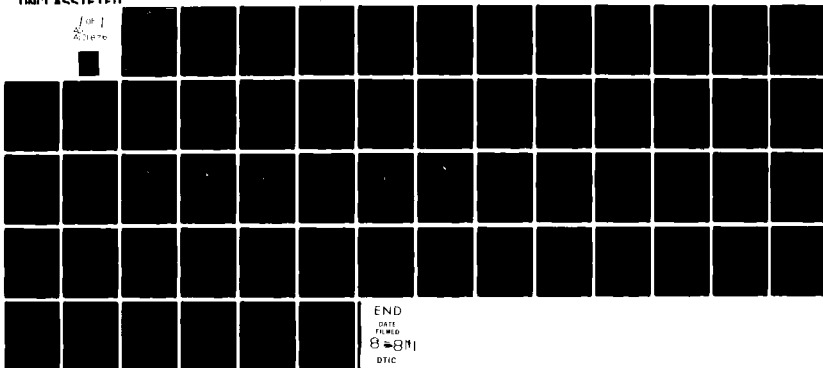
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## SUMMARY

The accuracy of sedimentary interval velocities derived from a towed hydrophone array can be predicted in the ideal case of horizontal interfaces and an idealized source pulse. The parameters of the prediction model are the two-way normal-incidence travel time and RMS velocity to the horizon of interest, array length, and source pulse width. First, the range of RMS velocities likely to be encountered in the upper 500 m of sediments is determined. Second, the error in these RMS velocity measurements is calculated as a function of array length, height above the sea floor and pulse width. Third, the RMS velocity error is converted to interval velocity error as a function of layer thickness and velocity, pulse width, and array length and height. It should be emphasized that these errors do not include any additional errors generated by the hyperbolic travel-time assumption and do not include the degradation encountered for dipping layers. In addition, our discussion refers to the analysis of one Common Shot Point or Common Depth Point Gather. Considerable improvement in an average interval velocity determination can be obtained by the analysis of several nearby CSP's or CDP's. This improvement will depend on lateral continuity for the horizons of interest. In the discussion which follows, we emphasize that only one CDP or CSP is being considered.

A short pulse, long array, small height above the sea floor all will contribute to increased interval velocity

resolution. A 1 km long array and a source with a 2 millisecond pulse width, both towed 100 meters above the sea floor, can yield interval velocities with errors of 2 percent for beds extending from 50 to 500 m below the sea floor. For beds thinner than 50 meters the error increases rapidly, becoming approximately 10 percent for 20 m beds. For beds greater than 200 m thick whose tops are 400 meters below the sea floor the error is 8 percent. Again the error increases rapidly for thinner beds. For an array altitude of 500 m the minimum error is 7 percent and this increases rapidly for beds thinner than 150 m.

A 1 km long array, 2 millisecond source pulse, and a tow altitude of 100 m is recommended to approach the desired 1 percent error criterion on interval velocity. These requirements are within current technological and ships operational capabilities. A deep towed array longer than 1 km might prove difficult to deploy, tow accurately and safely, and retrieve. A pulse width of 2 milliseconds is a compromise between the opposing requirements of velocity resolution and depth of penetration. A tow altitude of 100 m might be maintained in areas of known topography without the risk of dragging the source and array along the bottom. Operational considerations might alter the above array and source specification, but the resultant change in interval velocity accuracy can be predicted.

The 1 km array should consist of 25 hydrophones (or hydrophone groups) spaced 40 meters apart. The sampling interval required is 0.5 millisecond per hydrophone with 12 bit accuracy. This yields a data rate of 600 kilobits/second (exclusive of engineering data). The relative altitude of the individual

hydrophones with respect to the source should be known to 1 m and deviations of the array from the horizontal (kiting) should be minimized. Because CDP gathering of the data is recommended the altitude of the array should not change rapidly over a period of 15 minutes. This source and receiver array towed near the bottom will yield considerably more accurate interval velocities in thinner beds than those obtainable with a surface source and array.

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## 1. Array Velocities

Subsurface interval velocities are typically derived from array velocity measurements. Thus, the accuracy of interval velocity measurements are dependent on the accuracy of the determination of array velocities and event arrival times. To determine the proper array configuration for a deep towed source and receiving array, when the objective is to determine accurate interval velocities to a depth of 500 meters beneath the sea floor, we have investigated the RMS velocities expected for various array altitudes above the sea floor. (Array velocities are usually equated with RMS velocities even though this is only the case when there is no dip.) To generalize the analysis we have used an interval velocity function which varies linearly with depth. That is

$$v = a + bz.$$

Since RMS velocities are dependent on the actual interval velocities and hence the geology, the use of a linear interval velocity depth function where the slope,  $b$ , varies from 0 to 2.5 km/sec/km, will include most sedimentary cases of interest. In Figure 1, we display the array velocities derived by integrating this linear interval velocity depth function. In Figure 1a, for example, the array is at an altitude of 100 m above the sea floor, and array velocities were calculated for linear interval velocity functions where the slope increased from 0 to 2.5

km/sec/km in increments of .1 km/sec/km. The asterisks on the array velocity functions indicate depths of 500, 1,000, 1,500, 2,000 and 2,500 m respectively. At the recommended altitude of 100 m the array velocity varies from 1,500 to 3,000 m/sec, indicating that a broad suite of array velocities are available from which interval velocities may be derived. (The upper limit is 2,000 m/sec if we consider only the upper 500 m of sediments.)

In Figure 1b the array altitude was increased to 250 m and the suite of array velocities available has decreased. In Figure 1c, which corresponds to an array altitude of 500 m, we see that the range of array velocities available has further decreased. In fact, at a tow altitude of 500 m all the array velocities for a reflector 500 m beneath the sea floor are within the region 1.5 to 1.8 km/sec. To compare the type of array velocities that would be available from a conventional surface source and receiver array, we have continued calculating array velocities for an array altitude of 2,000 and 4,000 m above the sea floor (see Figures 1d and 1e). In both cases the suite of array velocities available decreases still further. In particular, when the array altitude (water depth for a surface array) is 4,000 m all of the interval velocities for a subsurface reflector 500 m below the sea floor have RMS velocities that range between 1.5 and 1.6 km/sec. Thus, an interval velocity determination based on array velocity discrimination where the array is 4,000 m above the sea floor becomes exceedingly difficult.

Figure 2 is a detailed plot of array velocities

available for an array altitude of 100 m above the sea floor. In this plot the array velocities vary from 1.5 to approximately 2.0 km/sec. This will be the array velocity region of interest. Thus the problem of determining the optimum array design for a deep towed source and array at an altitude of 100 m above the sea floor corresponds to determining the resolution possible for array velocities from 1.5 to 2.0 km/sec for reflection times up to 1.0 second.



## 2. Array Velocity Resolution

To develop a measure for defining array resolving power, we have considered the case of perfect bandwidth. That is, all frequencies for a specified passband are equally excited. Since bandwidth is inversely proportional to the pulse duration we consider bandwidth as one requirement for determining the resolving power of an array for various array velocities. Other variables are array length, reflection depth and the actual array velocity.

Since array velocities are usually derived from an approximately hyperbolic travel time—distance relation it is common to search through the observed reflection travel time data on a trial and error basis for all reasonable hyperbolic travel time paths across the array. That is, the two way travel time,  $T$ , to a reflection event is defined as

$$T^2 = T_0^2 + X^2/V^2$$

where  $T_0$  is the normal two way travel time to the reflection event,  $X$  is the source-receiver offset and  $V$  is the RMS velocity as defined by Dix 1955, or in the case of dip as defined by Shah 1973. By scanning the observed array data for coherent arrivals at all the possible two way normal times and RMS velocities it is possible to derive an interval velocity function using just the coherent array arrivals and Dix's 1955 small angle formulation. Thus, before considering the question of interval velocity

resolution, we must first consider array velocity resolution. (We will assume that there is no dip and relatively small source-receiver offset so that the RMS and array velocities can be equated. Realizing, of course, that if both these conditions are not satisfied any interval velocity determination based on these assumptions will be in error.)

In the case of perfect bandwidth where the corresponding time resolution is one sample, a direct measure of array resolving power is the unit sample semblance statistic. Semblance, a widely used coherency statistic, is commonly used to determine array velocities and arrival times. Usually, semblance is defined as the sum of all possible cross correlations between the seismic traces for a trial x-t trajectory normalized by the auto-correlations of each seismic trace. Thus, the definition of semblance is

$$S = \sum_w \left( \sum_i x_i \right)^2 / N \sum_w \sum_i x_i^2$$

where the  $x_i$  are the data samples across the array for a trial trajectory and N is the number of seismic traces. In this definition the summation over the correlation window, w, placed outside both the upper and lower sums is used to increase the energy and therefore the statistical stability. To define array resolving power we will not include these additional summations. Rather, we consider that the observed data consists of perfect delta functions in time and that array resolving power can be based on a unit sample semblance coefficient above a certain threshold. This unit sample semblance is a particularly

interesting measure of coherency since it is normalized between zero and unity (unity being the case of perfect coherence) and can be related directly to the variance of the distribution.

That is, semblance is equal to

$$S = 1 - \frac{v}{\mu_2}$$

where  $v$  is the variance and  $\mu_2$  is the second moment of the distribution. Therefore, knowing the value of semblance we can in fact infer the variance and hence standard deviation of the distribution.

We consider array velocity resolution for the same two way travel time,  $T_0$ , as the ability to discriminate between array velocities above and below the true velocity. We note that for velocities close to the true velocity, many of the small offset arrivals will be time coincident for sampled data. Only at some offset,  $x$ , will the hyperbolic travel time paths diverge from the true path. At this point, these time samples will no longer contribute to the unit sample semblance statistic and we will begin to discriminate between these array velocities. For analysis purposes we have used the semblance value of 0.5 as a measure of discrimination. This implies that we can discriminate between coefficients of 1 and 0.5. Using this definition the problem of defining array velocity resolution is reduced to determining when half of the reflection arrivals fall off of a trial hyperbolic trajectory. Clearly this is an idealized case and degradation due to additive noise or non-

perfect bandwidth will decrease the array resolving power. However, this definition will serve as a guide to determining the proper array configuration for a given bandwidth. For a fixed array length RMS velocity resolution could of course be improved by a large offset between the source and the array. However, the accuracy of the vertical incidence arrival time would decrease. Since interval velocity is calculated from vertical incidence arrival times as well as RMS velocities, it is unlikely that interval velocity resolution would improve by offsetting the source and the array.

Our decision to use a semblance value of 0.5 as a measure of discrimination is arbitrary, but suitable for the following analysis. The actual array length required will then be equal to twice the offset at which the arrivals first begin to fall off the trial trajectory. (Any other semblance value would imply an array length of 1/S of this value.) Semblance as a measure of coherency is also sensitive to the number of estimates available. For example, in the noise free case if only two estimates are available (two channels), the semblance statistic is a poor discriminator since the only possible values are 0, .5, and 1. As the number of channels increases the discrimination capability of semblance improves. (Where noise alone is present, no coherent arrivals, we expect the summations in the numerator and denominator to be equal and the limiting semblance value would be 1/N.) Thus, if a higher semblance value is used for a basis of discrimination, the number of channels that would be required should also increase to maintain the comparable level of discrimination.

To determine the offset where the arrivals begin to diverge in the case of perfect bandwidth, we take the partial derivative of time with respect to array velocity for the standard hyperbolic travel time equation and solve this equation for the array size necessary for a specified percent error,  $E$ , in array velocity. That is,

$$X = \frac{V}{BE} \sqrt{\frac{1}{2} + \sqrt{(T_0 BE)^2 + \frac{1}{4}}}$$

where  $T_0$  is the two way normal time,  $B$  is the bandwidth and  $V$  is the array velocity. In Figures 3, 4, and 5 we have displayed the array size necessary for a specified RMS velocity error at vertical two way travel times of .133, .333, and 1.333 seconds below the array for bandwidths of 100, 200, 500 and 1,000 Hz based on a semblance coefficient of 0.5. (In these plots the array length calculated from the above formula was doubled to determine the actual array length required.) In Figure 3a we display the array size necessary for a vertical two way travel time of .133 second and a bandwidth of 100. To achieve a one percent error in array velocity determination, an array size of 3.9 km is required for an array velocity of 2.0 km/sec. In Figure 3b the bandwidth increases and an array size of about 2.1 km is required to achieve the same percent array velocity resolution. In Figure 3c, where the bandwidth has now become 500 Hz it is possible to achieve the same array resolution with an array size of 1.0 km. In Figure 3d where the bandwidth is 1,000 Hz

we see that it is possible to achieve the same percent error with an array size of approximately 0.6 km. Thus, as the bandwidth increases the required spatial aperture decreases. Clearly it is desirable to tow the minimum array length and to have as broad a bandwidth as possible. However, achievement of the necessary bandwidth will be restricted by the spectral characteristics of available sources.

In Figures 4 and 5 we perform the same analysis but the time to the reflection horizon is increased. As expected, when the time to the reflecting horizon beneath the array increases the required bandwidth and/or the array size must increase to achieve the same accuracy in velocity resolution.

Based on Figure 2, we expect the array velocities to be between 1.5 and 2.0 km/sec for times of approximately 1 to 1.5 seconds beneath the array corresponding to depths of 500 m below the sea floor for reasonable interval velocity functions. In addition, we expect that it is possible to obtain source bandwidths on the order of 200 or possibly even 500 Hz. This would indicate, based on Figures 3b and c, that an array size of 1 km will be adequate for resolving array velocities to within one percent for the expected arrival times and for velocities associated with sedimentary horizons up to 500 m below the sea floor. Clearly, if more bandwidth can be obtained and the array size increased, the resolution will also increase. The bandwidths we have considered are idealized in the sense that we assume that all frequencies are equally excited within this band. A bandwidth

of 500 Hz actually requires a digitization rate of at least 1,000 Hz, or for a bandwidth of 200 Hz a digitization rate of 400 Hz.

### 3. Spatial Aliasing

Spatial aliasing is encountered when the sampling interval in space is too coarse for a given phase velocity across the array. For example, in Figure 6 we plot frequency versus wave number and have indicated a phase velocity of 2 km/sec. We have also indicated the Nyquist wave numbers corresponding to receiver separations of 100, 50, 33 and 25 m. Whenever energy is traversing the array at a phase velocity of 2 km/sec it will be aliased and appear to lie along the lines slanting upward to the left. For instance, at 25 m spacing a 2 km/sec phase velocity is aliased above 40 Hz. This would be a problem if frequency-wave number processing was to be performed on the original array data. Usually this is not the case. It is common practice to scan the array arrivals only in a specified array velocity band since one knows approximately the array velocities to be determined. Thus, even though the data may be aliased it is often unimportant in practice. If one were to process the original array data in the frequency-wave number domain (and were to discriminate based on phase velocity) the alias lines would have to be followed in the manner indicated in Figure 6.

Dense spatial sampling is required to remove aliasing. In Figure 6 a spatial sampling interval of 10 m is required to have unaliased data above 100 Hz. Since this necessitates increasing the number of channels, it becomes difficult to achieve this type of spatial resolution. The exploration



industry is rapidly moving towards longer arrays with denser spatial sampling and this has necessitated recording on such media as video tape because of the correspondingly high data rates. Day to day exploration, however, is commonly carried out with 100 or 50 m array element spacing intervals. While higher spatial resolution is clearly desirable, this would increase the number of channels and require significantly greater digitization bandwidth than is currently available.

#### 4. CDP Versus Common Shot Data

Array data is acquired in a Common Shot Point (CSP) mode, that is, all channels are recorded for a given shot. In exploration, this data is reordered into the Common Reflection Point or Common Depth Point (CDP) mode. CSP and CDP are equivalent geometries in the case of no dip. In the presence of dip, however, the CDP geometry has significant advantages. Basically this geometry averages the ray paths such that the RMS velocity determined from the array velocity can be used to give a better indication of the subsurface interval velocities. Several papers have been written, for example, Shah 1973, which indicate how array velocity determinations can be turned into true RMS velocity determinations even in the presence of dip for the CDP geometry. Thus, by measuring the time dip on a normal incidence record section and using array velocities one can correct for the presence of dip to improve the interval velocity determination. In addition, in most cases the effect of modest dips on the CDP geometry are quite minor. CDP geometry has a significant advantage over CSP geometry for horizontally discontinuous or rough reflectors. In the CSP mode an interface would have to persist laterally for at least one half the array length in order that true reflections from it be recorded on all hydrophone groups. In the CDP mode this persistence requirement is reduced to the smear in the CDP caused by errors in the Shot Point placement (navigation) and reflection dip. This advantage becomes important when attempting

to measure RMS velocities from rough interfaces such as oceanic basement, since one need only find a short segment where basement is smooth and horizontal. Additionally, after applying moveout corrections and stacking the section will be a better representation of the sub-sea-floor geology in the CDP case.

To acquire CDP data using a 1 km deep-towed array with 25 channels and 40 m spacing would require a 20 m shot spacing interval for full 25 fold CDP coverage. If the array were towed at 1 kt (.5 m/sec) then a 40 sec repetition rate would be required. It would be preferable to tow the array at 2 kt or 1 m/sec and in this case a 20 sec repetition rate would be required. The CDP mode requires accurate positioning of the shot points to avoid smearing the Common Reflection Points (CRP). For 25 fold data, 25 consecutive shots contribute to each CDP. It is the average spacing of the shots that must be controlled accurately, rather than the spacing between individual shots. For a 1 km array the 25 shots are spaced over a distance of 400 m. If one specifies a maximum smear in the CRP's, caused by misplacement of the shot points, of 5 m, then the average shot spacing must be accurate to 1 percent. It is assumed that longitudinal deformation or stretching of the array will be small (i.e.  $\sim 2$  m). If the array is decoupled from the source, variations in the distance between the source and the array must be minimized and known for each shot point. For data acquired at 1 kt it would take 1,000 seconds or 16 minutes to acquire one CDP. Therefore, only slow deformations of the array would be tolerable. At 2 kts, however, the data would be

acquired in 8 minutes and more rapid deformations could be tolerated.

To acquire CDP data with a deep towed array will require that kiting be kept to a minimum. The minimum requirements of 500 Hz bandwidth necessitates that time be known to less than a millisecond, that is, we must know the array height to within a meter. In one meter a pressure change of 1.5 psi occurs and although an absolute measurement of pressure is not necessary, relative pressure and depth should be recorded to the required accuracy. (An accurate calibration of the pressure sensing units would be necessary.)

Two corrections are required because of kiting. The first is a timing correction and the second is a spatial correction. The array appears smaller as the kiting angle increases, thus it is absolutely necessary that the array deformation be known and that this deformation be removed on a shot basis prior to the CDP gather. Instrumenting the array with pressure sensors that have been calibrated initially and have an accuracy to better than 1.5 psi should provide sufficient information to remove the effect of kiting. In addition, if the array is towed with a drogue and the array is neutrally bouyant and the source and array are mechanically isolated from the towing cable, the corrections after the initial settling should be a minimum.

## 5. Interval Velocity Error Estimation

In a previous section we discussed array velocity errors. For relatively small offsets and no dip they can be considered RMS velocity errors. Here we relate RMS velocity errors to interval velocity errors.

Interval velocities are calculated from pairs of RMS velocities and their associated vertical incidence two-way travel times via Dix's 1955 formula:

$$V_{INT} = \sqrt{\frac{V_2^2 T_2 - V_1^2 T_1}{T_2 - T_1}}$$

where  $V_2$  and  $T_2$  are respectively the RMS velocity and travel time to the bottom of the layer, and  $V_1$  and  $T_1$  refer to the top of the layer. Each of these terms has an error associated with it. We know how to calculate RMS velocity errors and we assume that travel-time to a reflector can be determined to an accuracy of one source pulse width (i.e. the reciprocal of the bandwidth). From known errors in  $V_1$ ,  $T_1$ ,  $V_2$  and  $T_2$  there are several ways to estimate the error in  $V_{INT}$ .

One could assume that the errors in each of the terms add algebraically. In this case we would add percent errors during a multiplication or division and add actual error for additions or subtractions. The square root requires halving the percent error. This is a form of worst-case error estimation, since we assume that each term is in error by the maximum amount and in the most harmful direction.

A second method assumes that the errors add vectorially. Instead of adding errors (either percent or actual) we assume that they are at right angles to each other. Thus an error of 3 percent and an error of 4 percent would yield a combined error of 5 percent ( $3^2 + 4^2 = 5^2$ ). This is a form of most probable error, since we assume that each term will not always be in error by its maximum amount, that is the errors are independent of each other.

We have chosen a third method which is also a form of worst-case error estimation. First we calculate  $V_{MAX}$  by assuming that  $V_2$  is larger than and  $T_2$  is smaller than their true values by the maximum allowable error. Thus if  $V_2$  is 2,000 m/sec and the error estimate is 1 percent we set  $V_2$  equal to 2020 m/sec. Similarly if  $T_2$  is 1.0 second and the error is  $\pm 2$  milliseconds (1/500 Hz) then  $T_2$  is set to 0.998 seconds. In a similar manner we underestimate  $V_1$  and overestimate  $T_1$ . This will yield a large value of  $V_{MAX}$ , see Figure 7. We then calculate low values for  $V_2$  (1980 m/sec) and  $T_1$  and high values for  $V_1$  and  $T_2$  (0.998 sec). This gives us a low value of  $V_{MIN}$ . Then the percent error in  $V$  is estimated as:

$$E_v = \frac{V_{MAX} - V_{MIN}}{2 \cdot V_{INT}}$$

With this type of worst-case error estimation, the actual errors will be less than the estimated errors. However, we have prescribed error limits and the true interval velocity will be within these limits.

## 6. Interval Velocity Resolution

The above discussion gives a proper background to interpreting Figure 8. In Figure 8 we plot interval velocity percent error as a function of layer thickness ( $\Delta Z$ ). (It must be emphasized at this point that these errors do not include any error associated with assuming hyperbolic trajectories.)

Figures 8a to e show the interval velocity error as a function of layer thickness for a 1 km array towed 100, 250 and 500 m above the layer for bandwidths of 500 and 200 Hz. The six curves on each plot are for layer velocities from 1,500 m/sec to 2,000 m/sec in 100 m/sec steps. The material above the layer is assumed to have a velocity of 1,500 m/sec. The most important feature of these plots is the strong dependence of the error on array altitude and bandwidth. These emphasize the need for a large bandwidth source and a low towing altitude. Low altitude not only achieves higher velocity resolution for thick beds, but extends this high resolution to thin beds (down to 100 m in Figure 8a). It appears that 2 percent accuracy can be achieved with a bandwidth of 500 Hz and an altitude of 100 m. Two other features are evident; the relatively small effect interval velocity has on resolution, and in Figures 8a to 8d a slight decrease in resolution for thicker beds. This decrease in resolution results from flatter hyperbolic trajectories at the base of the bed as the time to the base increases. This flattening increases the error in the RMS velocity.

By way of comparison with a surface source and array we have included similar plots for arrays of 4.8 km and 10 km length in a water depth of 4 km. (Figures 8f and g) The 10 km array is hypothetical as we do not know of any in existence. Note that these two plots have a different thickness axis. For a 1 km thick bed the 4.8 km array has a minimum error of 3 percent and a 10 km array improves the accuracy to 1 percent. However, the velocity error increases rapidly as the bed thickness decreases below 500 m.

In conclusion we see that interval velocity accuracies near 2 percent can be obtained with a 1 km array towed 100 m above the sea floor with a source of at least 500 Hz bandwidth.



## 7. Refractions

Critical angle refractions or head waves might be observed with a 1 km array if it is towed close to the sea floor and there are shallow high velocity beds. These refractions would provide a useful measure of velocity at the top of the refracting horizon. Refractions normally observed with surface sonobuoys are generally low frequency which makes it difficult to accurately measure their arrival time across the array. The 1 km array coupled with accurate timing could yield good phase velocity estimates. However, in doing RMS velocity scans it may be necessary to mute or zero the large offset traces since these arrivals will lead to a degradation in the RMS velocities and thus the interval velocity determination.

### References

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Shah, Pravin M. 1973; Use of Wavefront Curvature to Relate Seismic  
Data with Subsurface Parameters, Geophysics, Vol. 38,  
pg. 812 to 825

### Figure Captions

Figure 1. Array velocities derived from linear interval velocity vs. depth functions for array altitudes of 100, 250, 500, 2,000, and 4,000 m. On each plot the normal two way travel times corresponding to depths of 500, 1,000, 1,500, 2,000, and 2,500 m are indicated. For an array altitude of 100 m the expected array velocities range from 1.5 to 2.0 km/sec and the expected two way normal times range from 0.5 to 1.0 secs. Increasing the array altitude decreases the suite of array velocities and increases the two way normal time. For an array altitude of 4,000 m all array velocities for a reflector 500 m below the sea floor are between 1.5 and 1.6 km/sec.

Figure 2. A detailed display of the expected array velocities for an array altitude of 100 m.

Figure 3. Array size (km) vs. percent array velocity error for a reflector at .133 sec of two way normal time (depth of 100 m for a velocity of 1.5 km/sec) for bandwidths of 100, 200, 500 and 1,000 Hz and for array velocities 1.5, 2.0, 2.5 and 3 km/sec. For a fixed two way normal time, as the bandwidth increases the percent array velocity error decreases.

Figure 4. Array size (km) vs. percent array velocity error for a reflector at .333 sec of two way normal time (a depth of 250 m for a velocity of 1.5 km/sec) for bandwidths of 100, 200, 500 and 1,000 Hz and for array velocities 1.5, 2.0, 2.5 and 3 km/sec.

Figure 5. Array size (km) vs. percent array velocity error for a reflector at 1.333 sec of two way normal time (a depth of 1000 m for a velocity of 1.5 km/sec) for bandwidths of 100, 200, 500 and 1,000 Hz and for array velocities 1.5, 2.0, 2.5 and 3 km/sec.

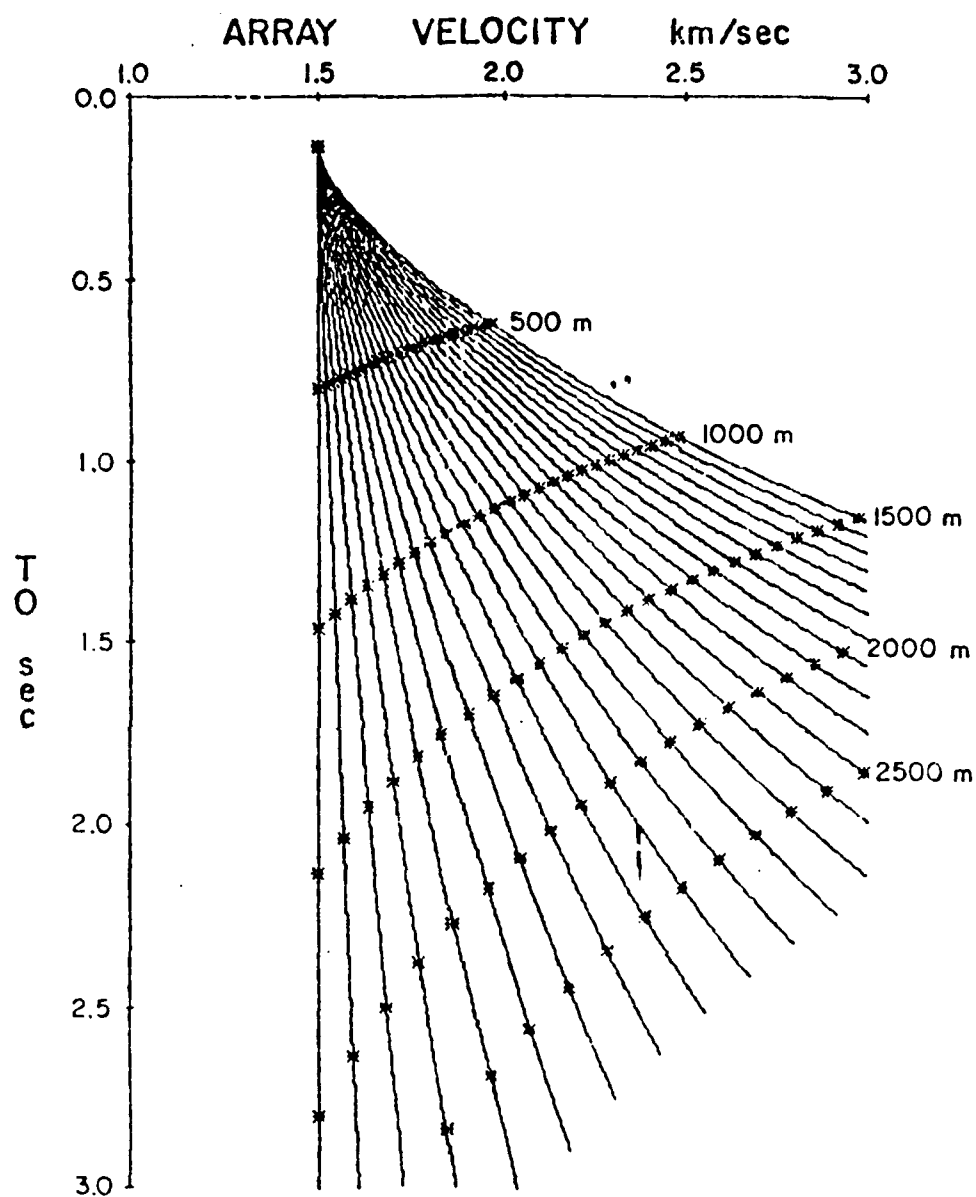
Figure 6. Frequency vs. wave number plot with a phase velocity of 2 km/sec indicated. The Nyquist wave numbers corresponding to spatial sampling intervals of 25, 33, 50 and 100 m are indicated. Whenever energy is traversing the array at a phase velocity of 2 km/sec it will be aliased and appear to lie along the lines slanting upward to the left. For the 2 km/sec phase velocity and a 25 m spatial sampling interval all frequencies above 40 Hz will be aliased.

Figure 7. Diagram indicating the method used to compute the percent interval velocity error from RMS velocity and normal two way travel time measurements. To determine the minimum interval velocity we overestimate the RMS velocity and underestimate the time at the top of the layer and underestimate the RMS velocity and overestimate the time at the bottom of the layer. To estimate the maximum interval velocity we underestimate the RMS velocity and overestimate the time at the top of the layer and overestimate the RMS velocity and underestimate the time at the bottom of the layer. Half the difference between these two interval velocities divided by the true interval velocity is the percent error used in Figure 8.

Figures 8a to c. Percent interval velocity error vs. thickness ( $\Delta Z$ ), for an array of 1 km towed 100, 250 and 500 m above the sea floor and a source bandwidth of 500 Hz. Each curve corresponds to an interval velocity of 1.5, 1.6, 1.7, 1.8, 1.9 and 2.0 km/sec respectively. As the array altitude increases the percent interval velocity error also increases. As the thickness of the beds decreases the percent interval velocity error increases significantly in all cases. As the thickness of the beds increases the percent interval velocity error eventually will increase because of the increased array velocity error.

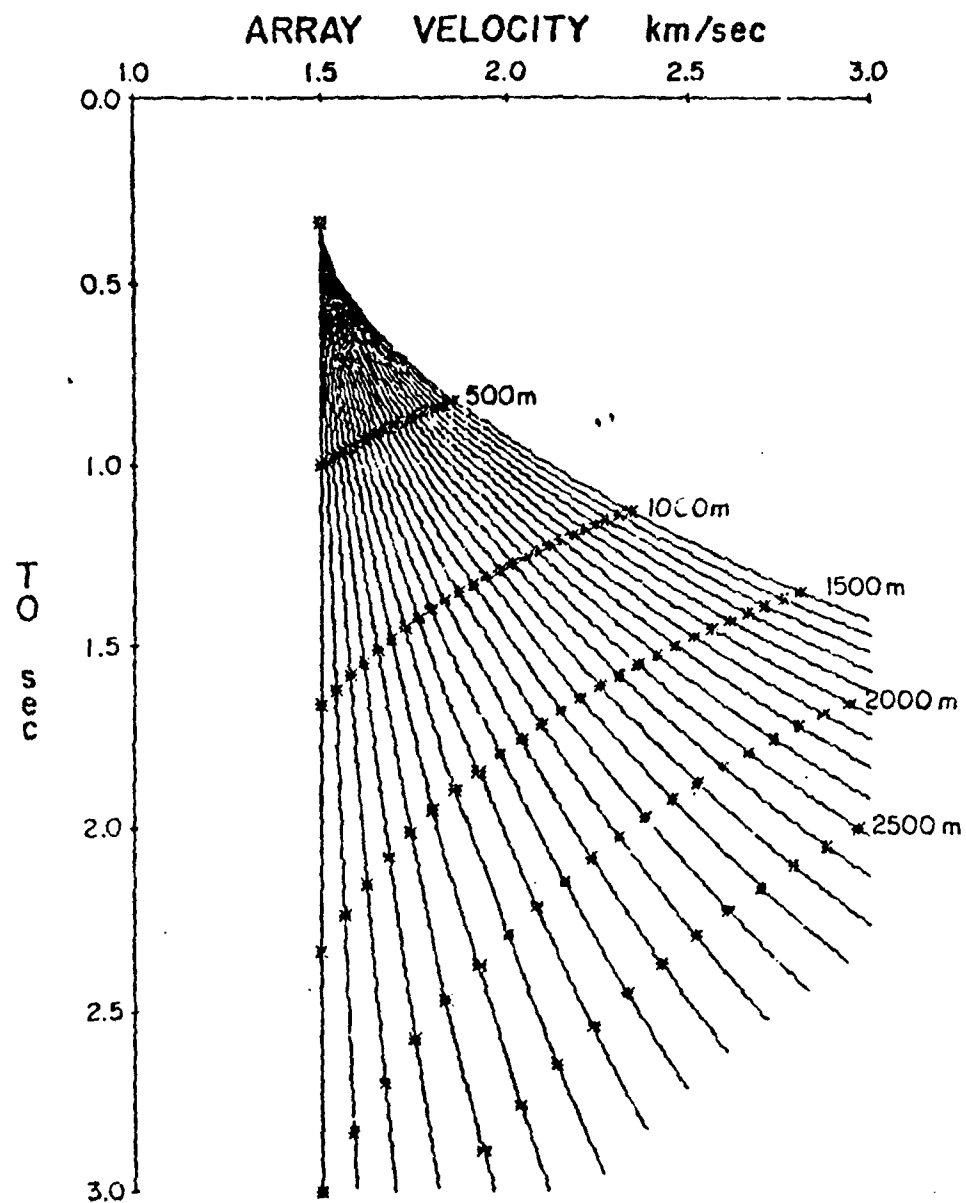
Figures 8d and e. Percent interval velocity error vs. thickness ( $\Delta Z$ ), for an array length of 1 km towed 100 and 250 m above the sea floor and a source bandwidth of 200 Hz. Each curve corresponds to an interval velocity of 1.5, 1.6, 1.7, 1.8, 1.9 and 2.0 km/sec respectively. In comparison with Figures 8a to c the decreased bandwidths result in significantly increased percent interval velocity error.

Figure 8f and g. Percent interval velocity error vs. thickness ( $\Delta Z$ ), for an array altitude of 4000 m, source bandwidth of 500 Hz and array lengths of 4.8 and 10 km. Each curve corresponds to an interval velocity of 1.5, 1.6, 1.7, 1.8, 1.9 and 2.0 km/sec respectively. For this array altitude (or for a surface array where the water depth is 4000 m) a 3 to 4 percent interval velocity error can be obtained for layers with a thickness of about 1 km for an array length of about 4.8 km. For an array length of 10 km the percent error decreases significantly. For both array



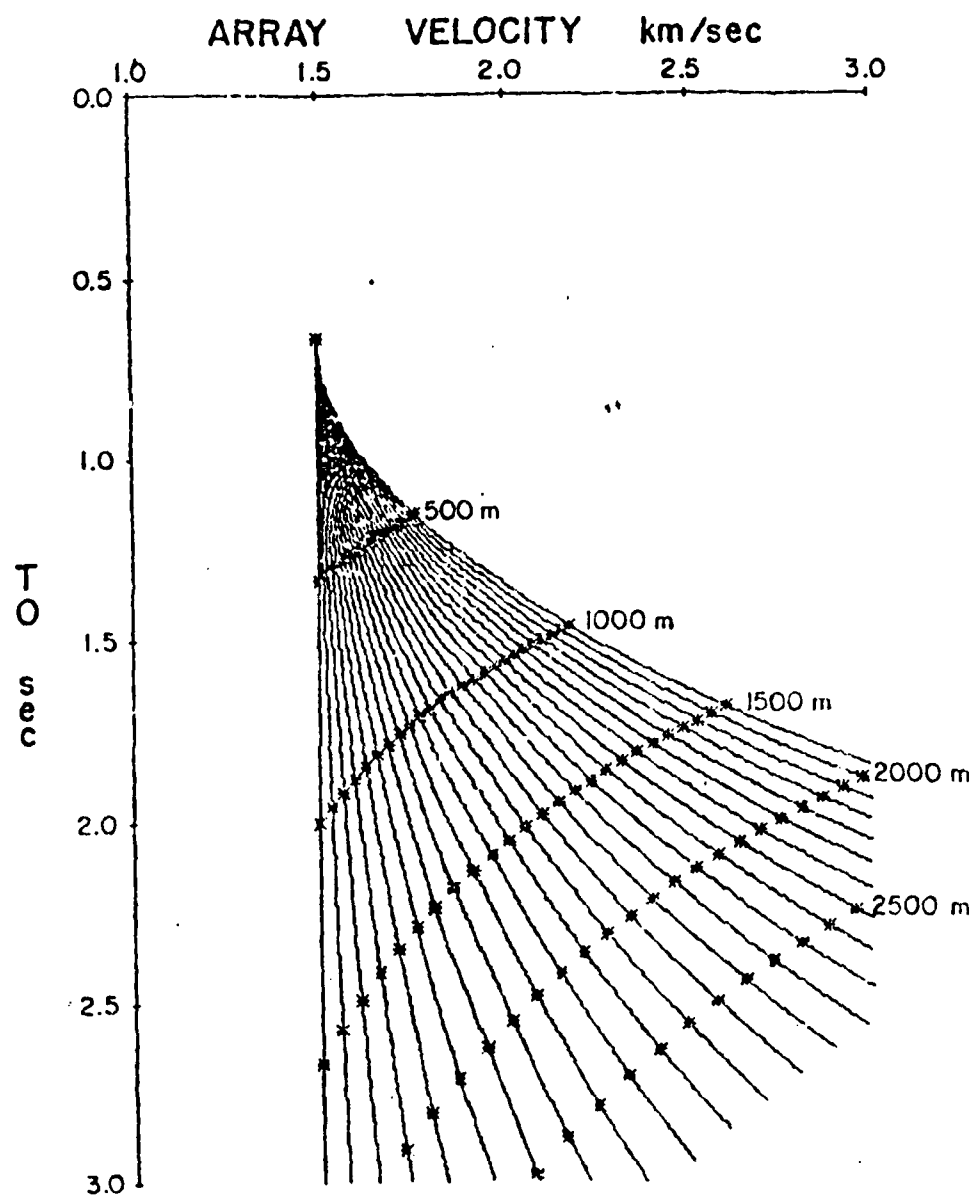
ARRAY ALTITUDE 100 m

Figure 1a



ARRAY ALTITUDE 250m

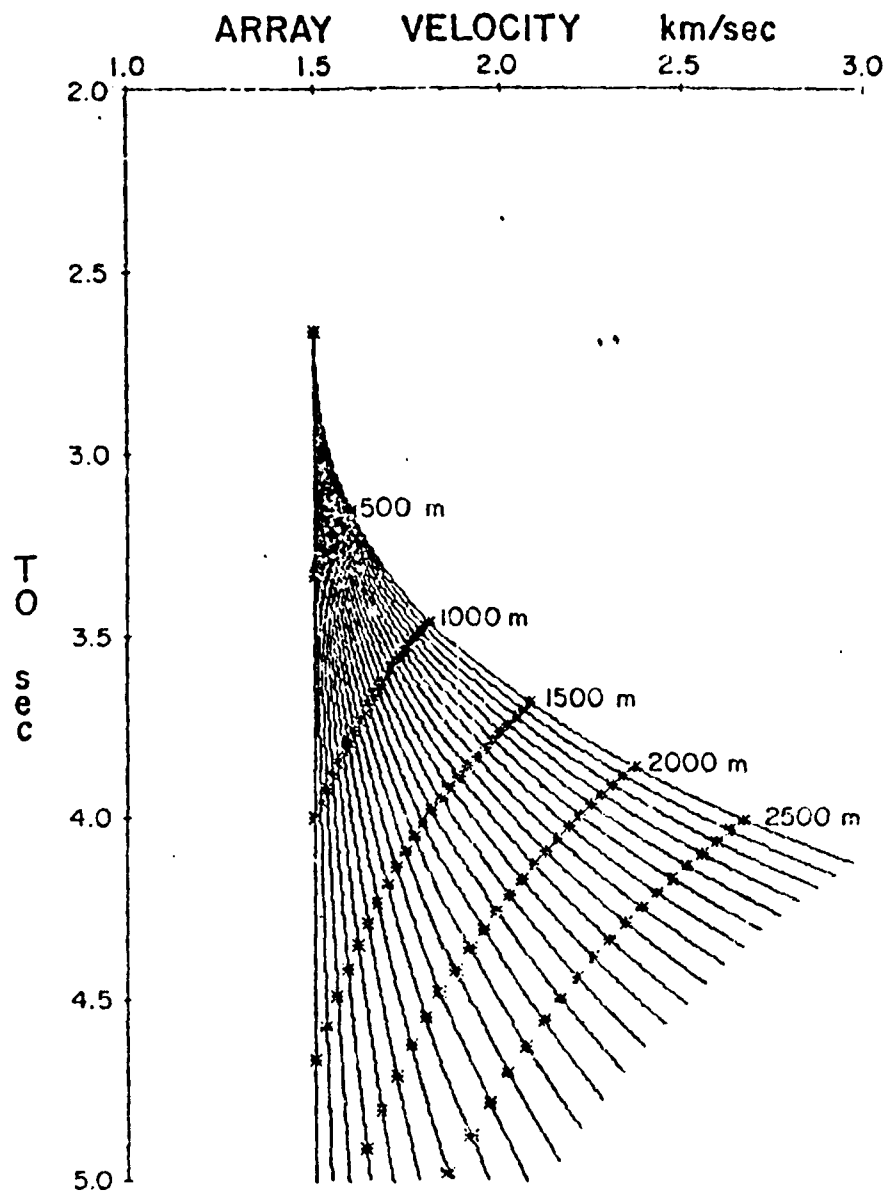
Figure 1b



ARRAY ALTITUDE 500m

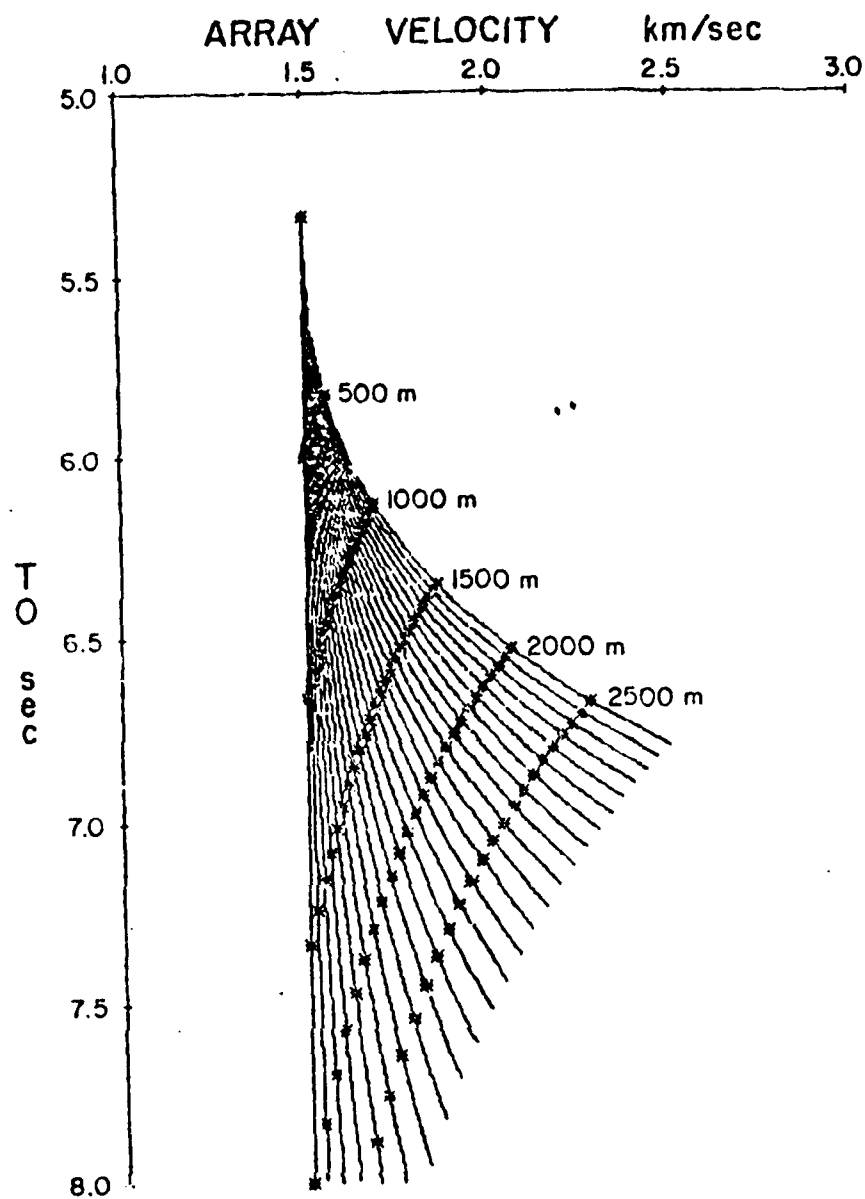
Figure 1c





ARRAY ALTITUDE 2000m

Figure 1d



ARRAY ALTITUDE 4000m

Figure 1e

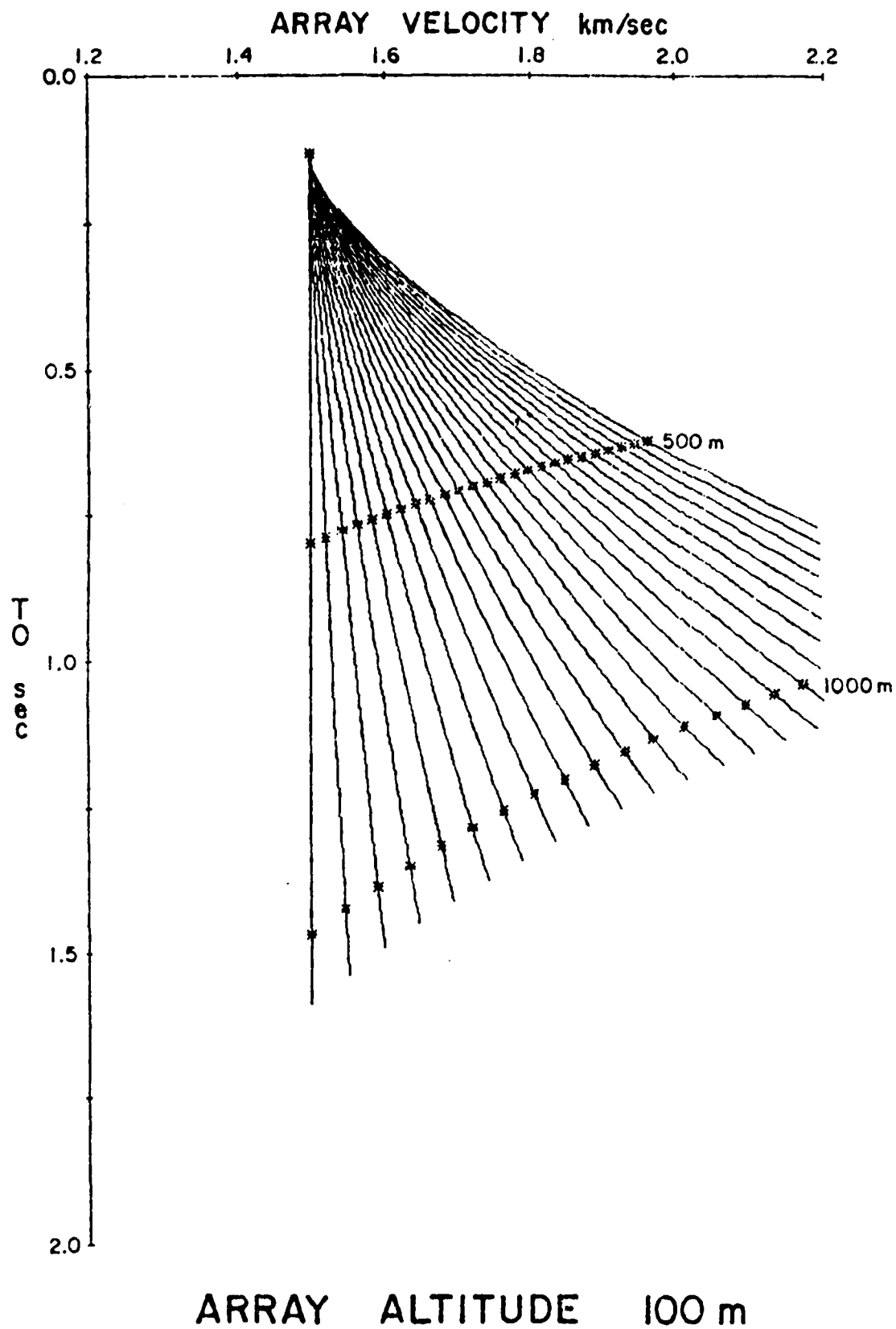


Figure 2

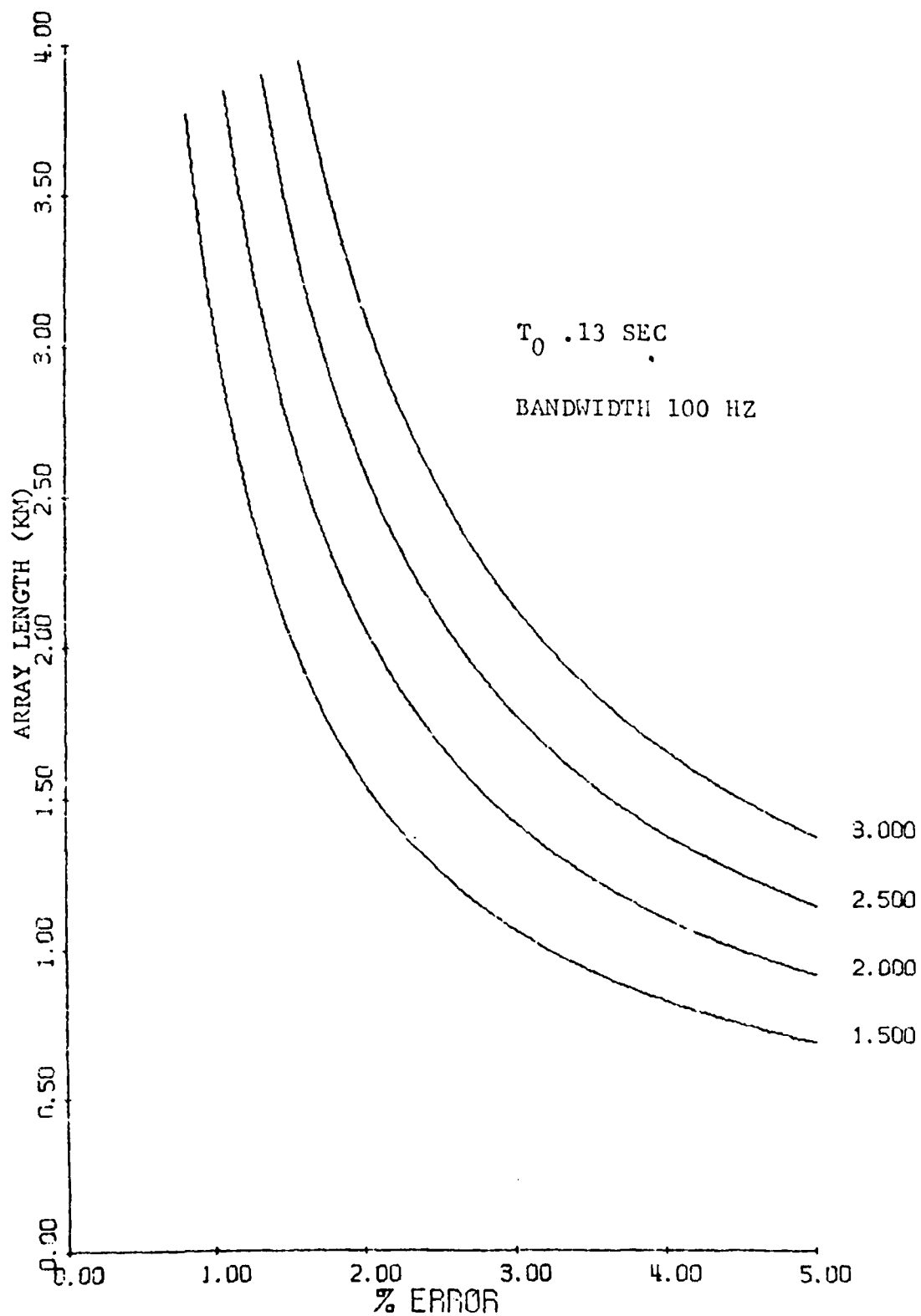


Figure 3a

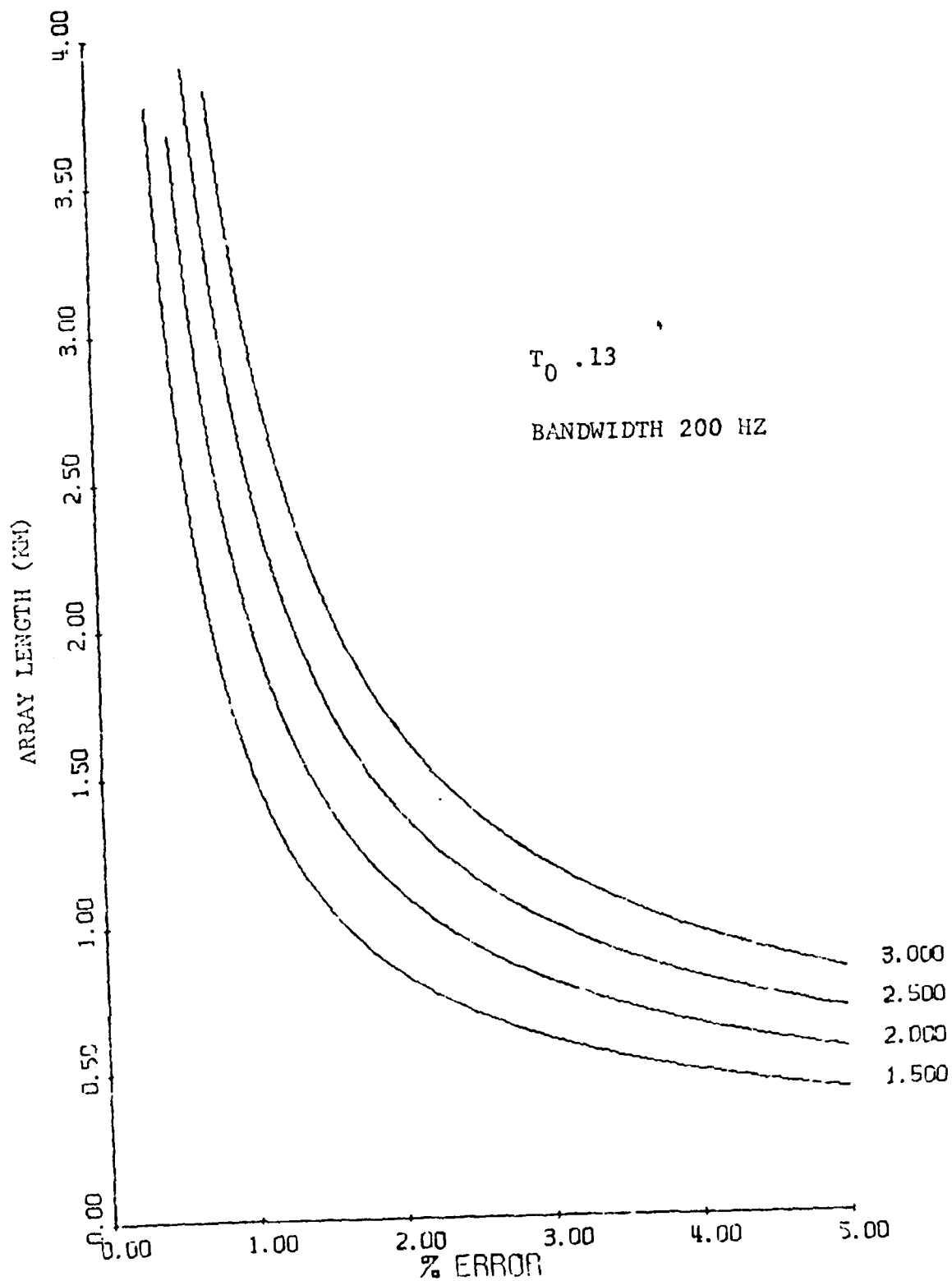


Figure 3b

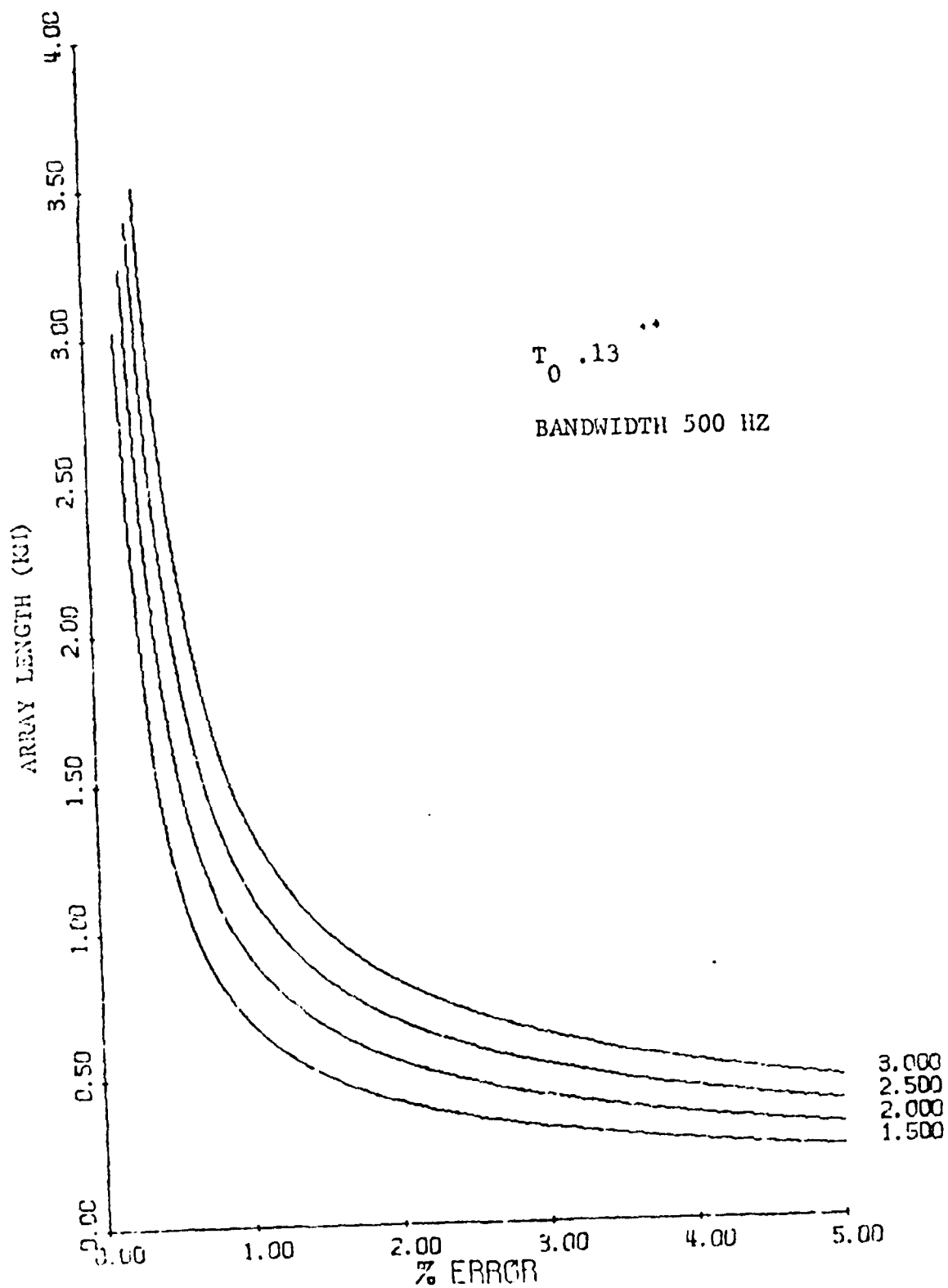


Figure 3c

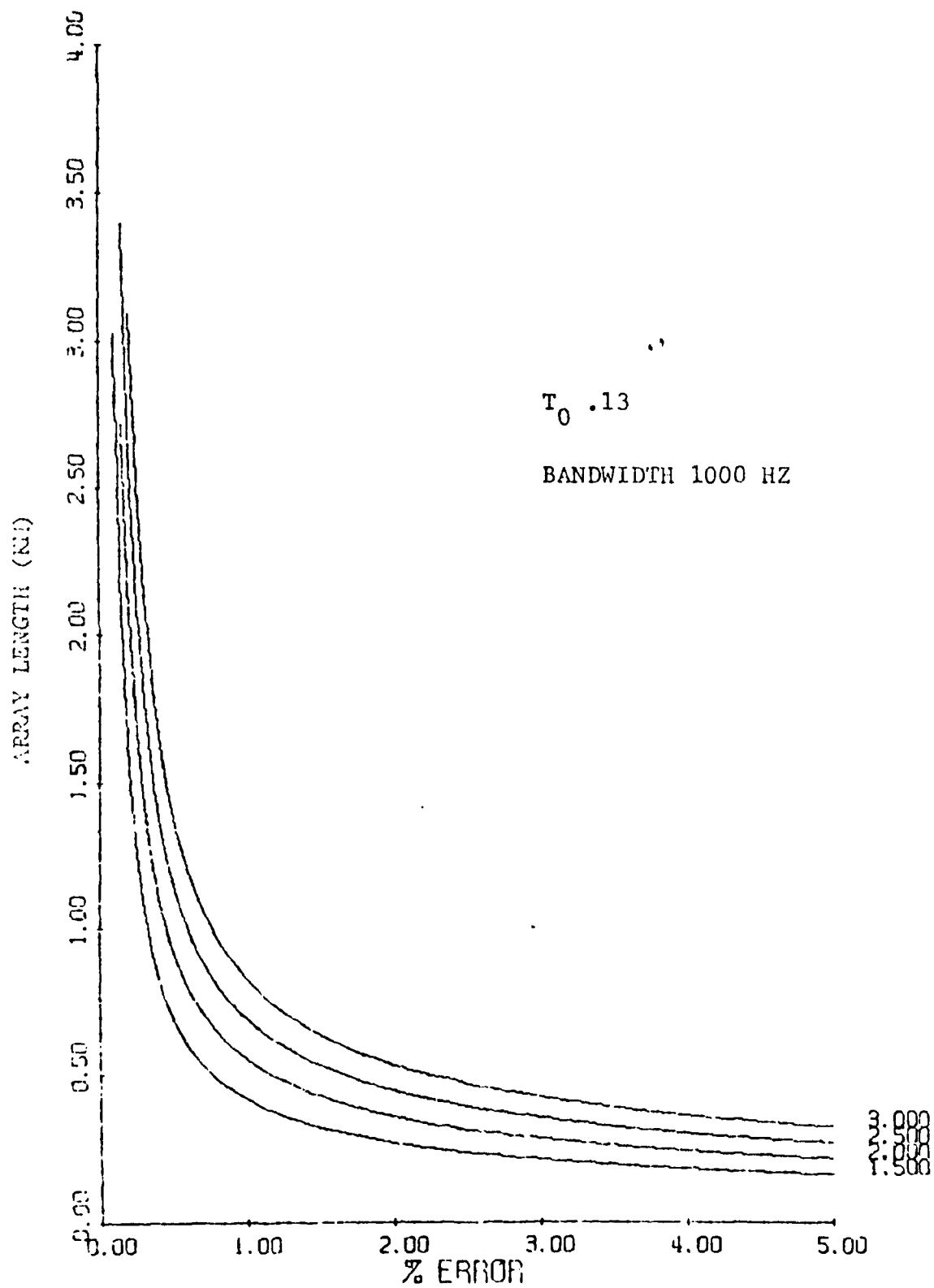


Figure 3d

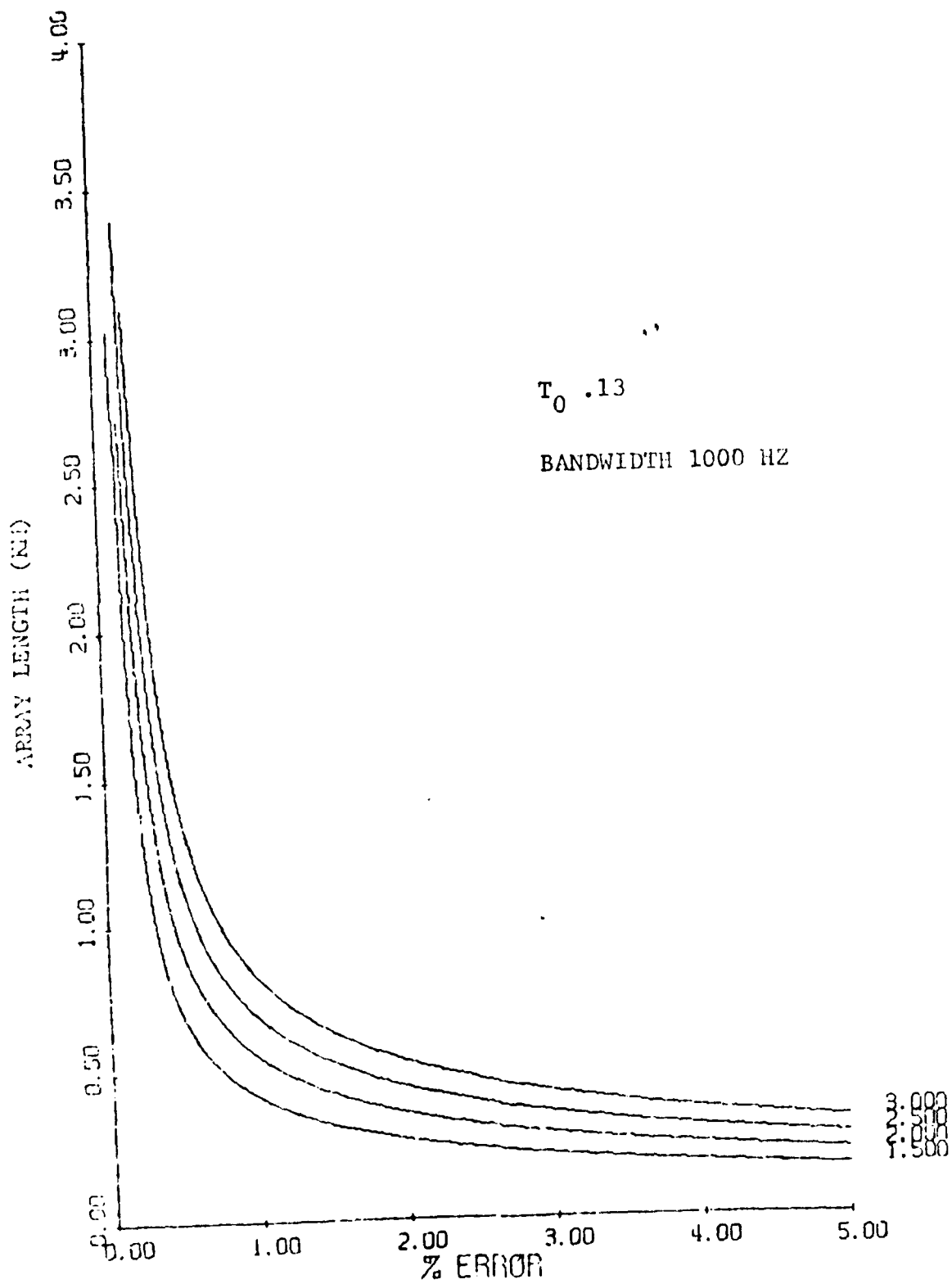


Figure 3d



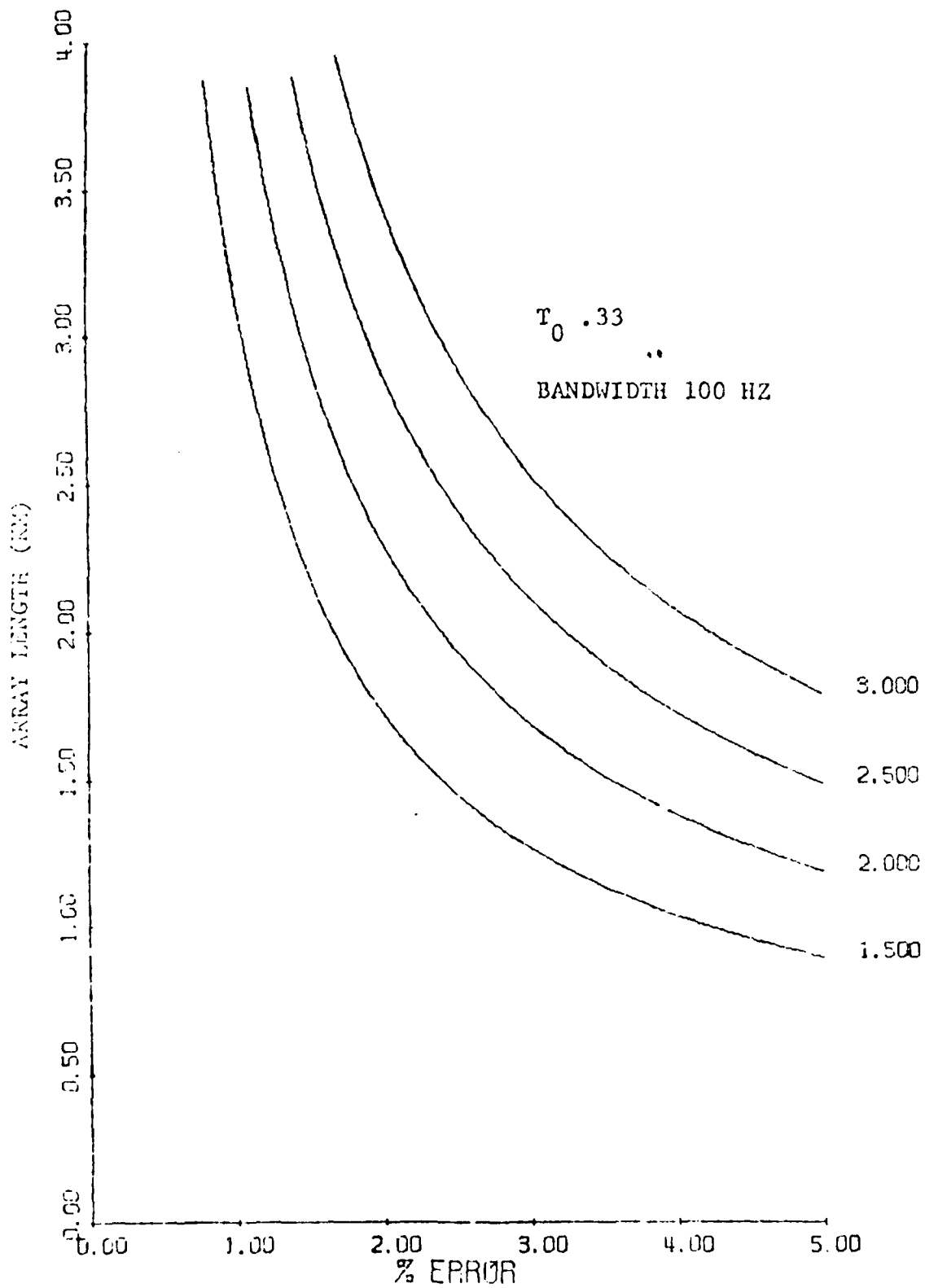


Figure 4a

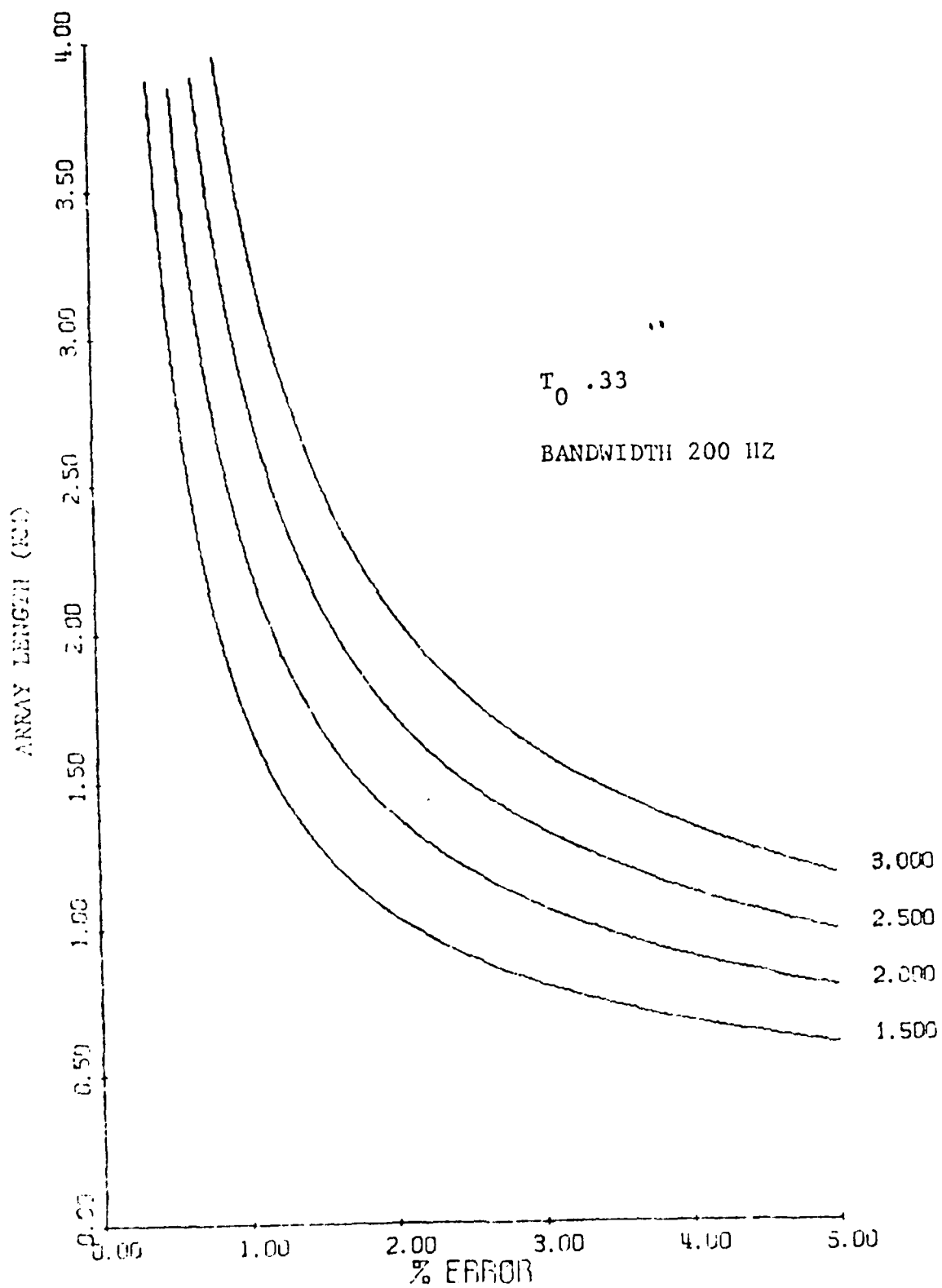


Figure 4b

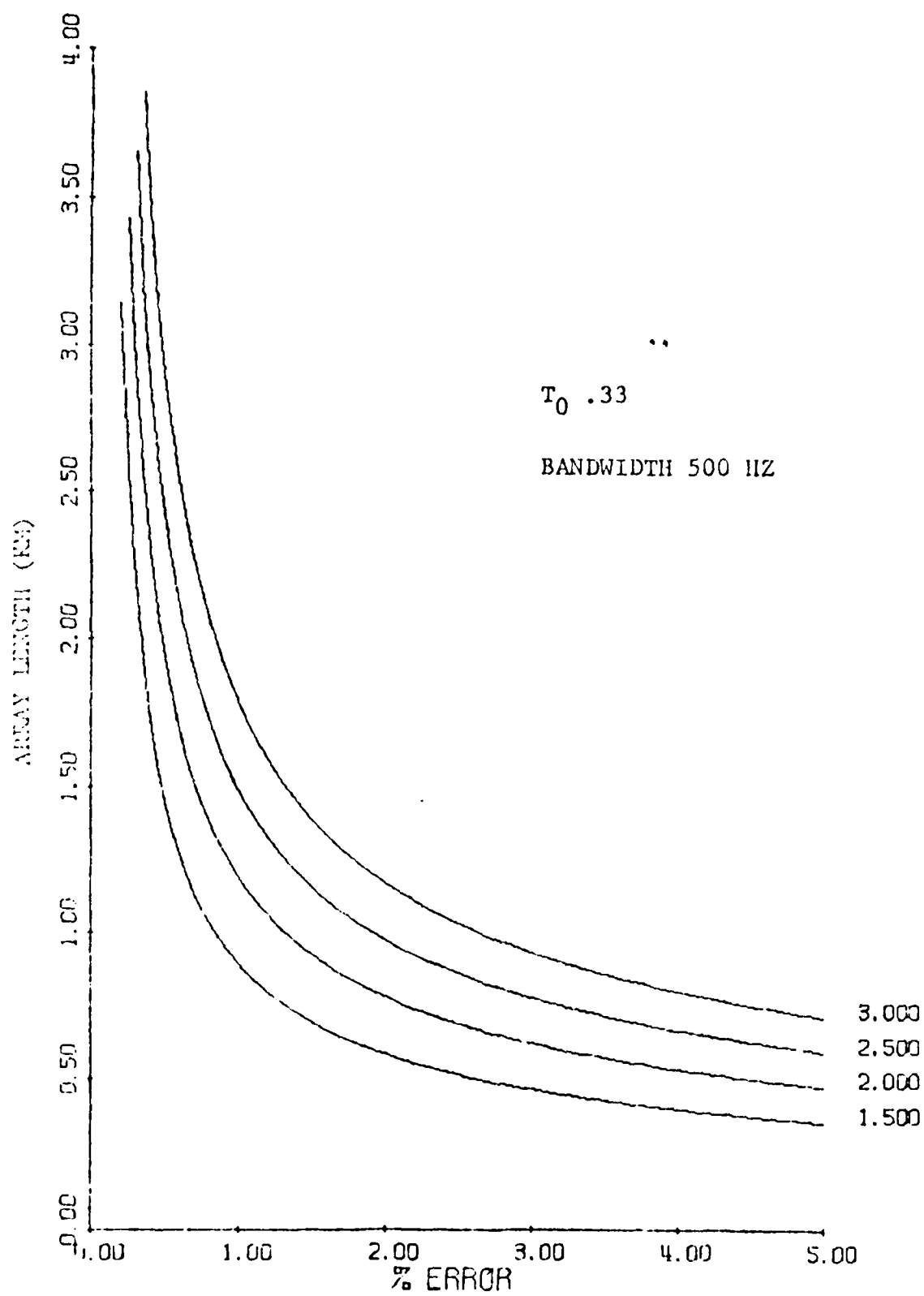


Figure 4c

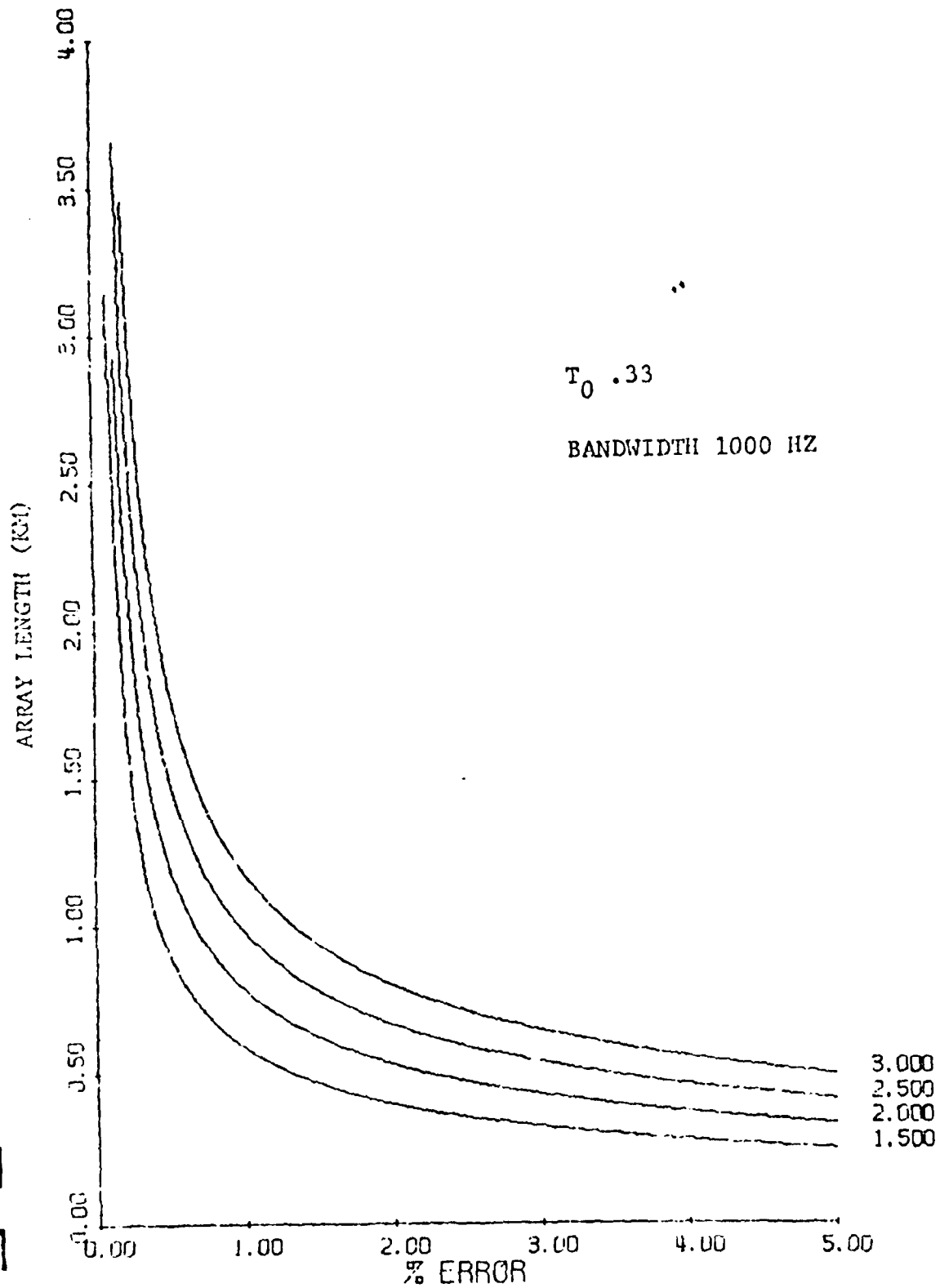


Figure 4d

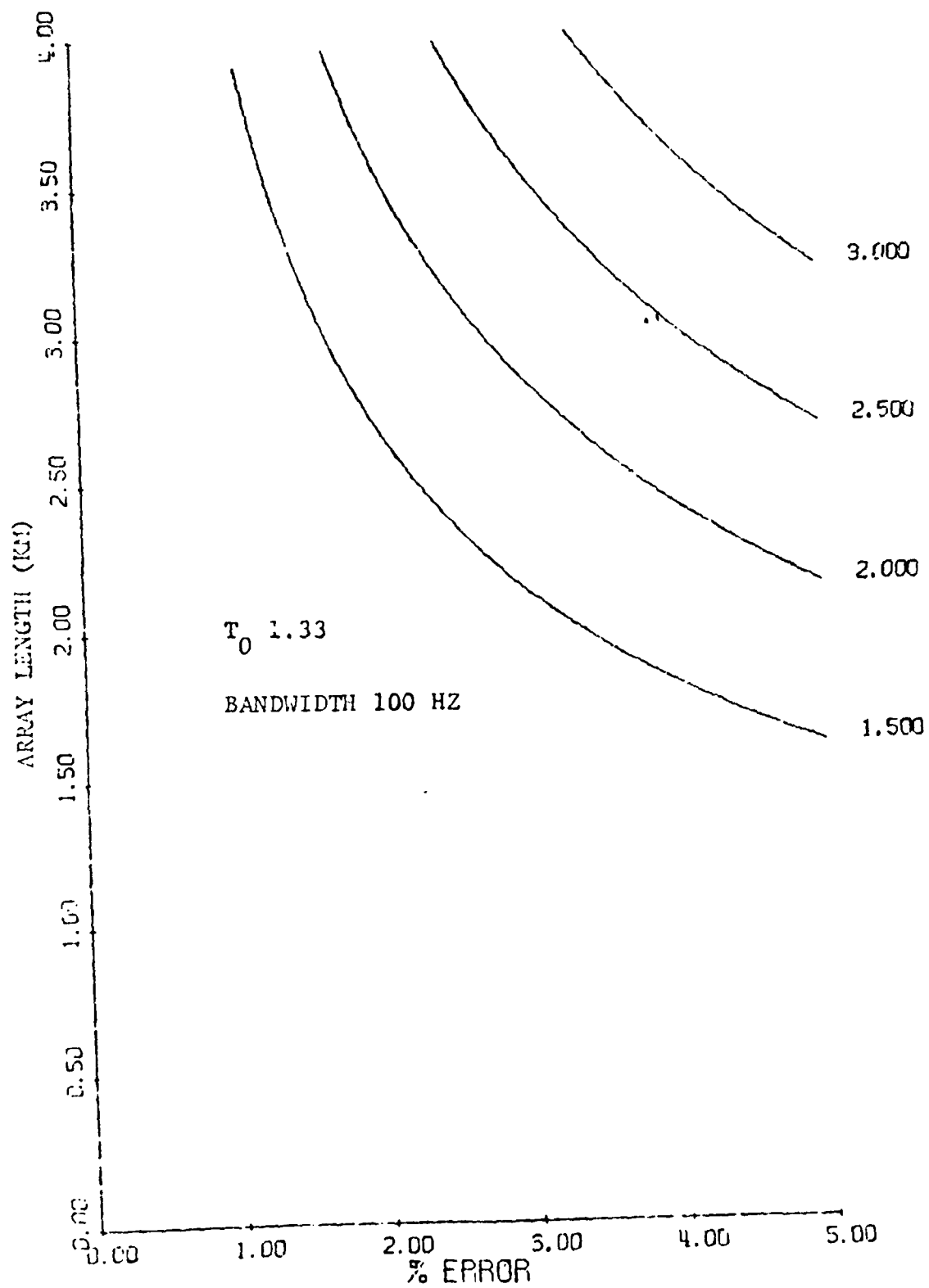


Figure 5a

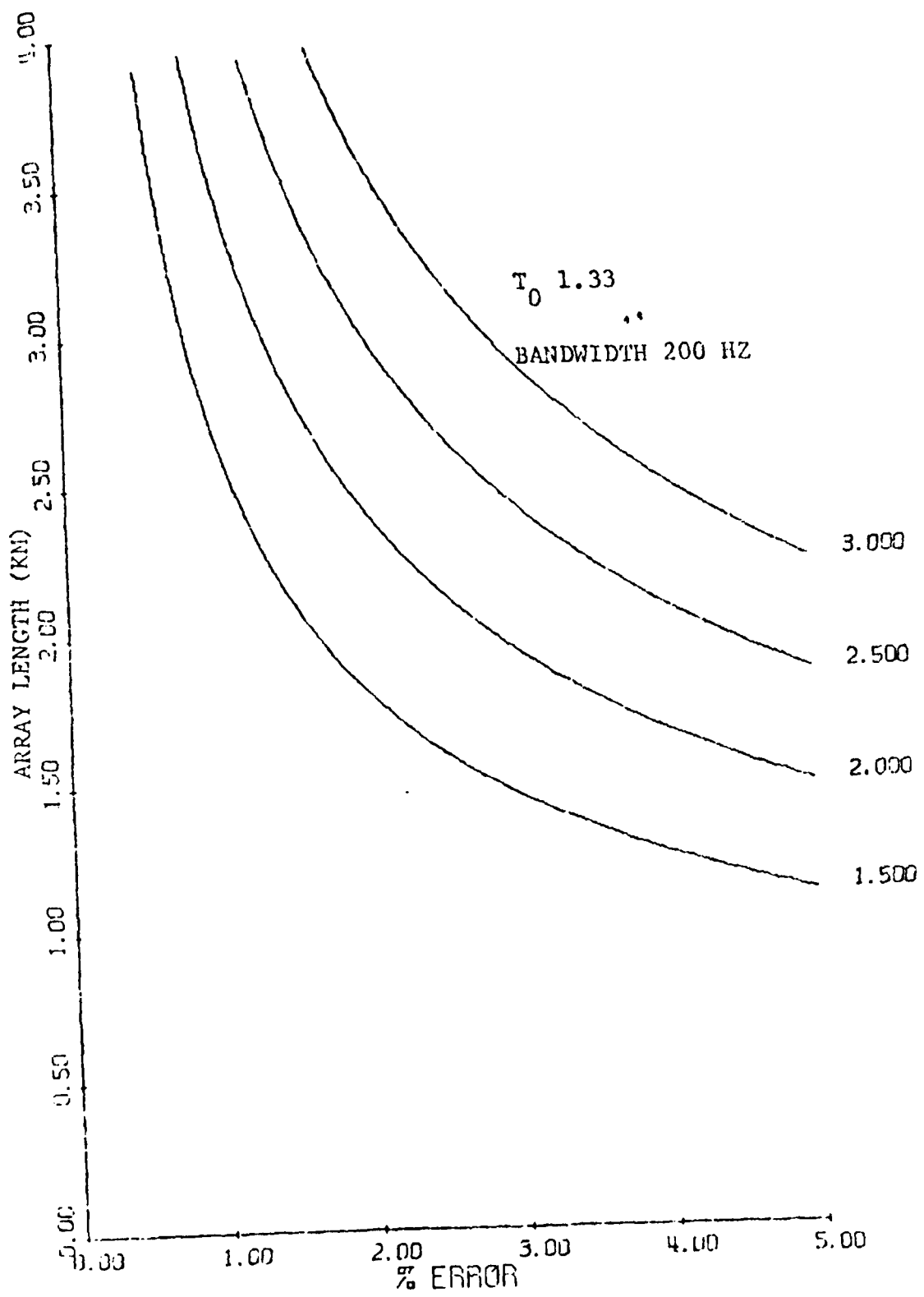


Figure 5b

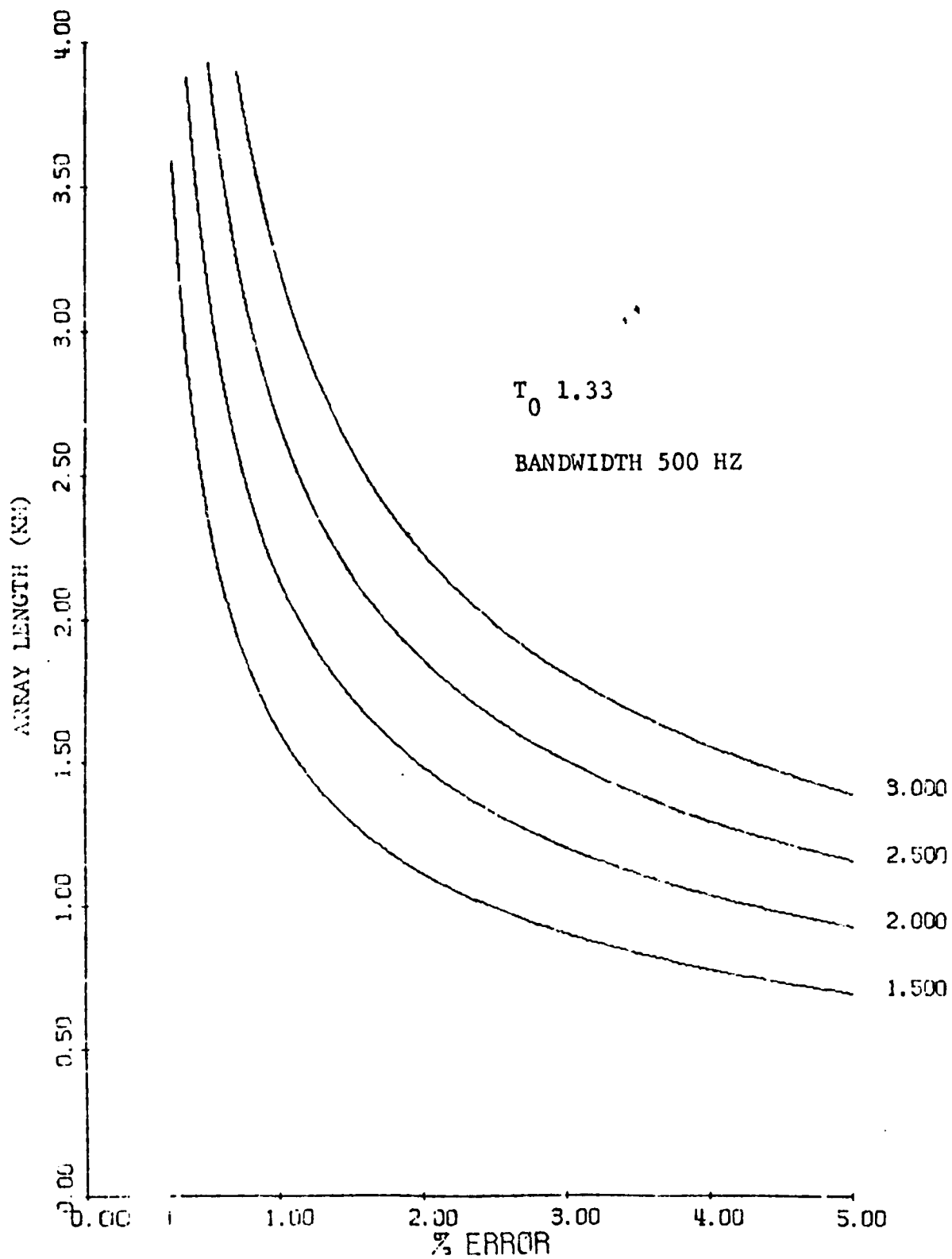


Figure 5c

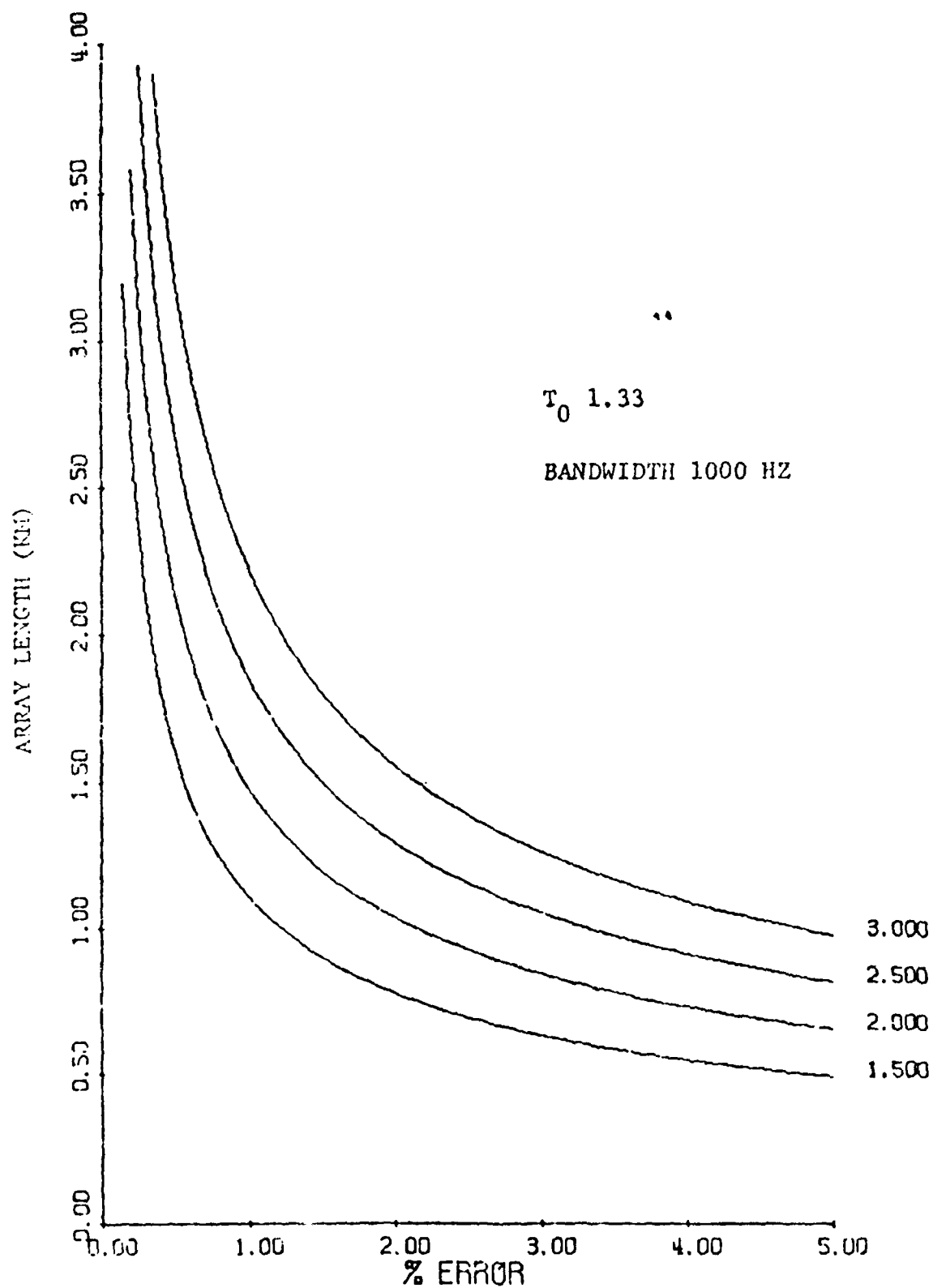


Figure 5d



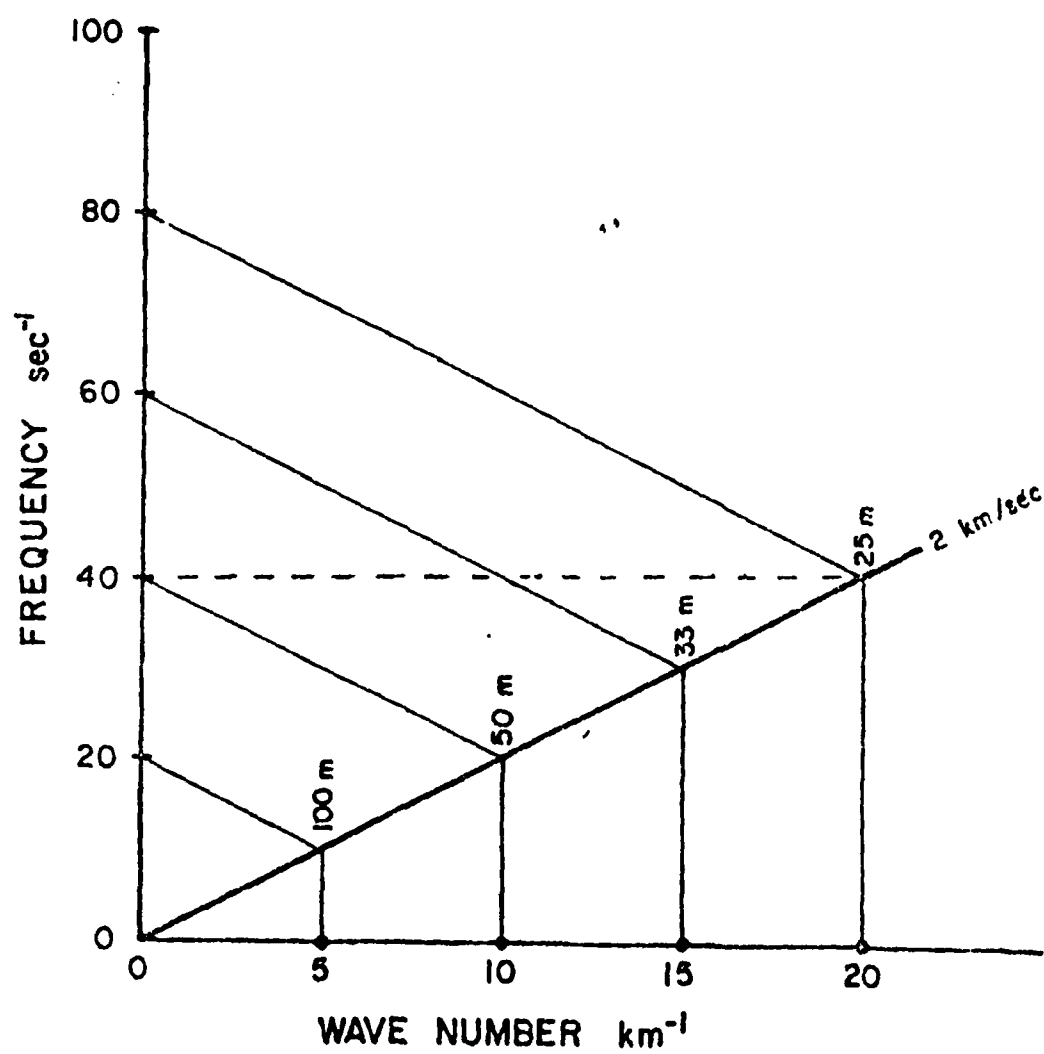


Figure 6

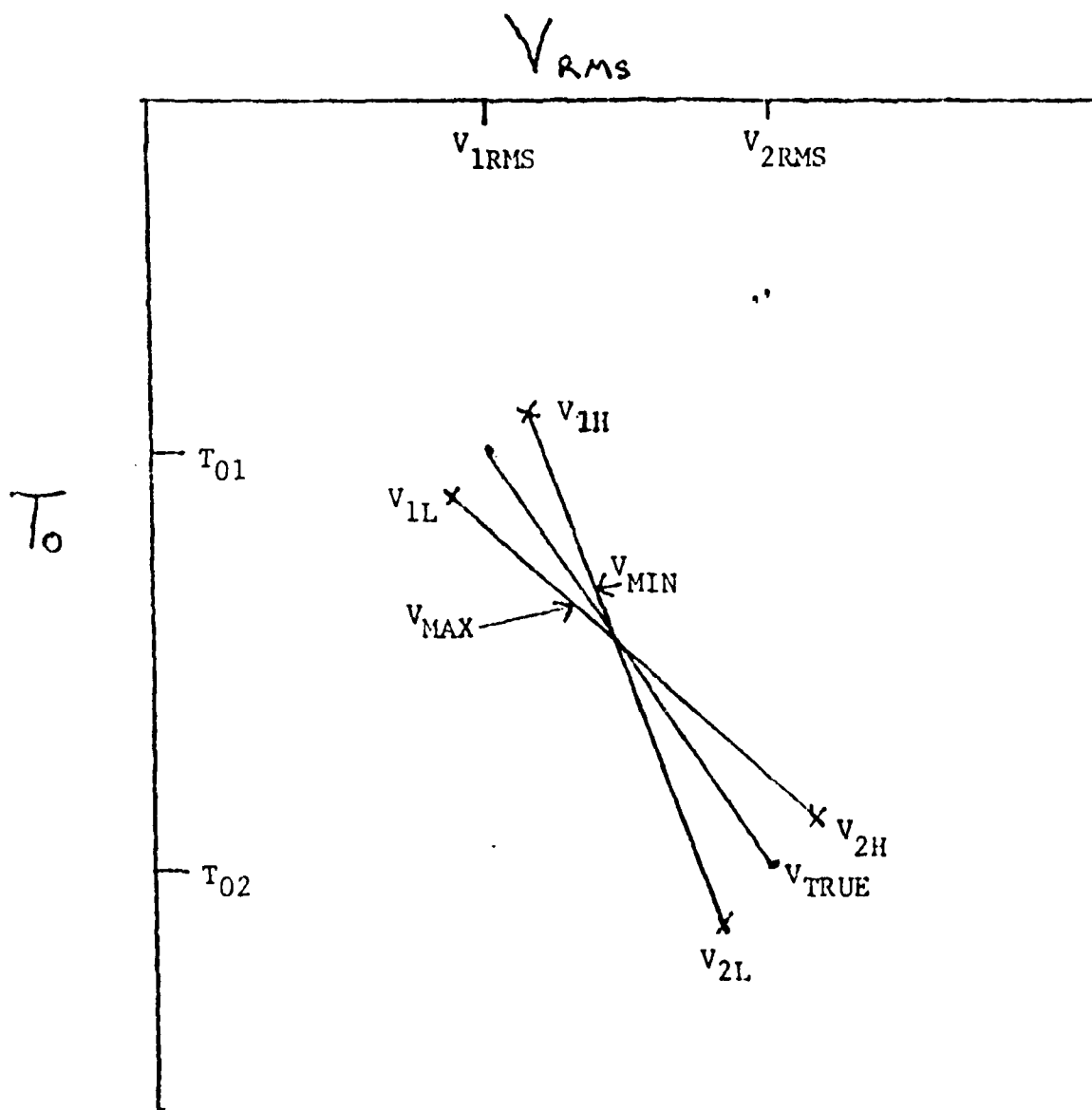


Figure 7

BANDWIDTH 500 Hz  
ARRAY ALTITUDE 100 m  
ARRAY LENGTH 1 km

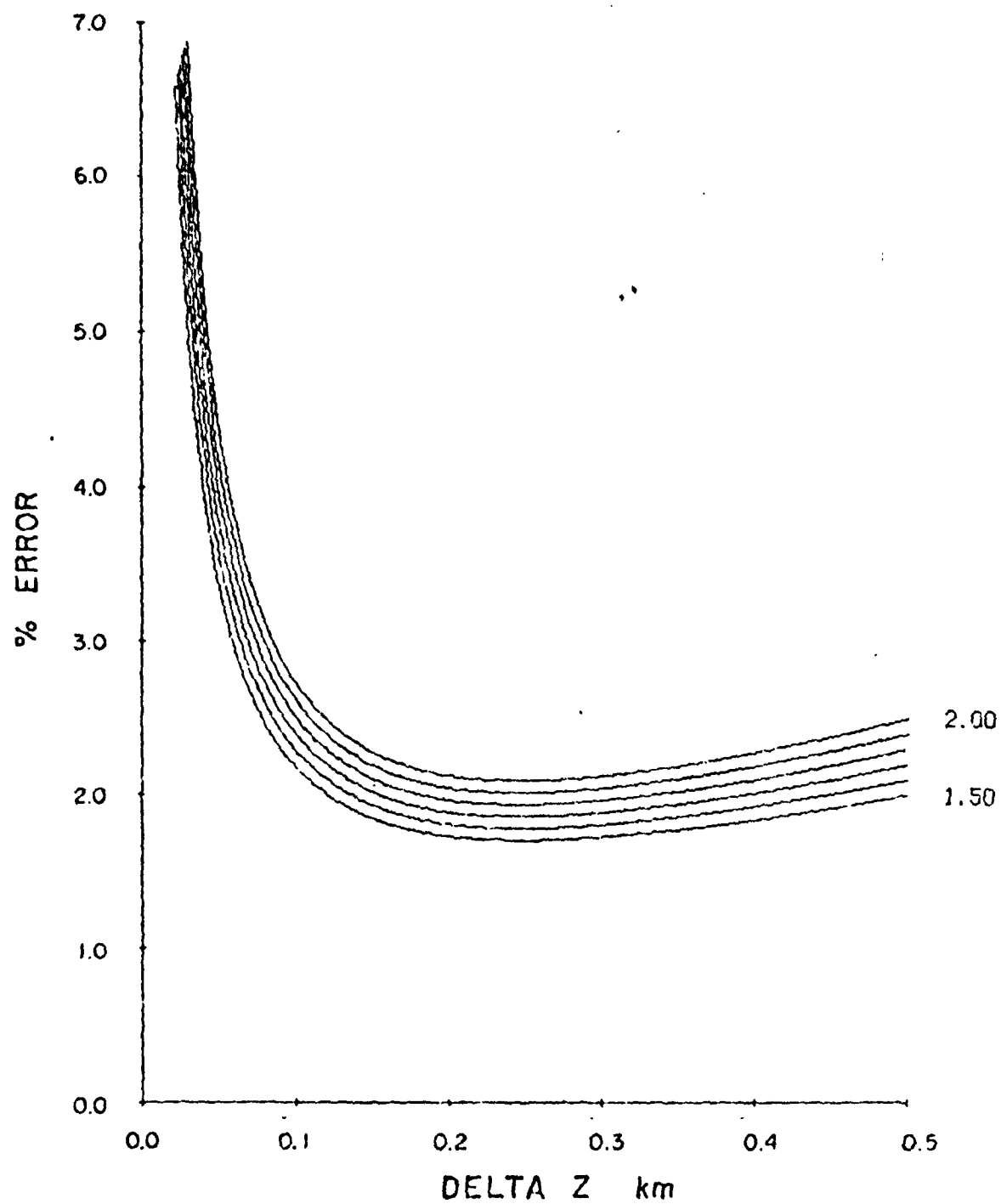


Figure 8a

BANDWIDTH 500 Hz  
ARRAY ALTITUDE 250 m  
ARRAY LENGTH 1 km

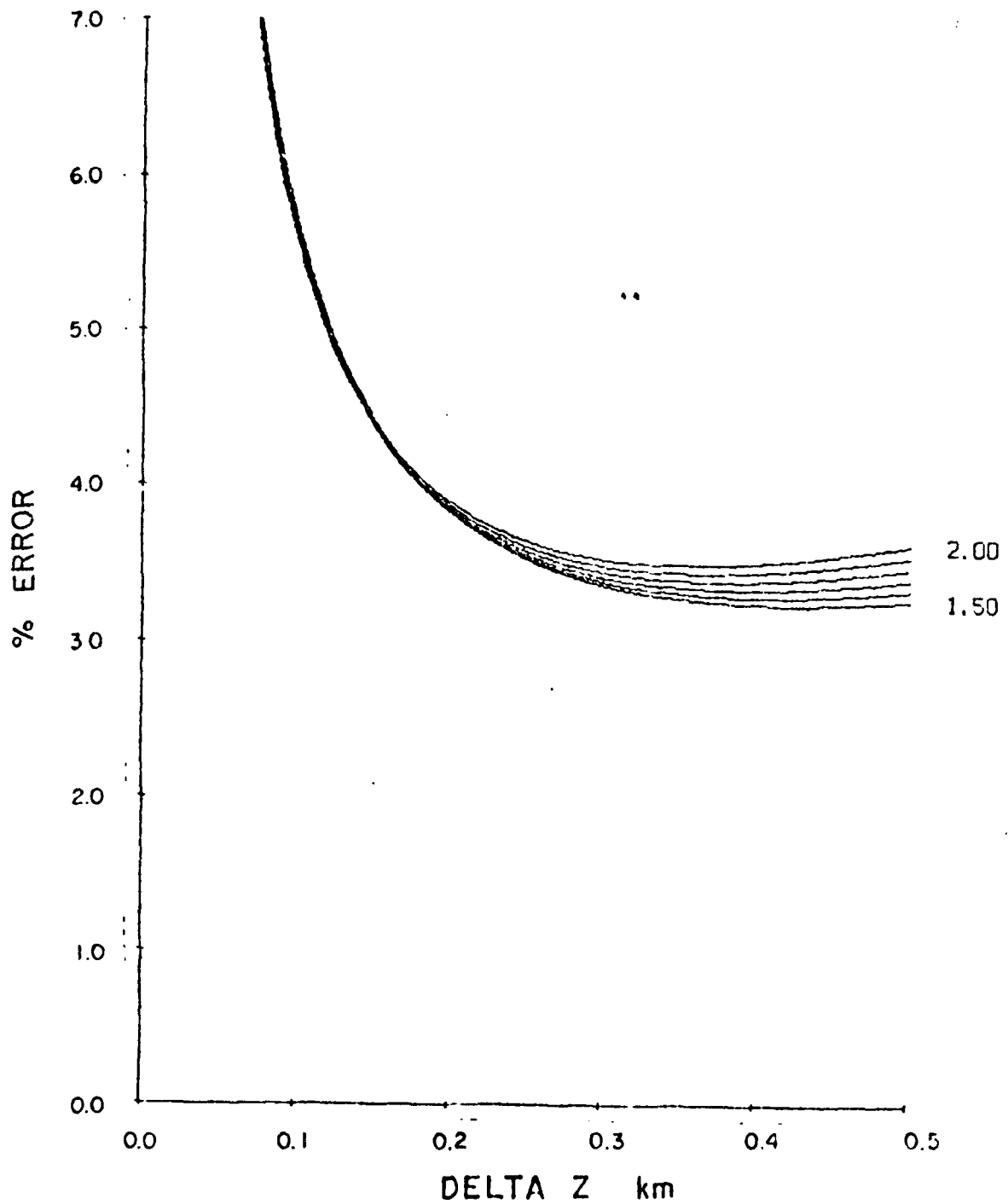


Figure 8b

BANDWIDTH 500 Hz  
ARRAY ALTITUDE 500 m  
ARRAY LENGTH 1 km

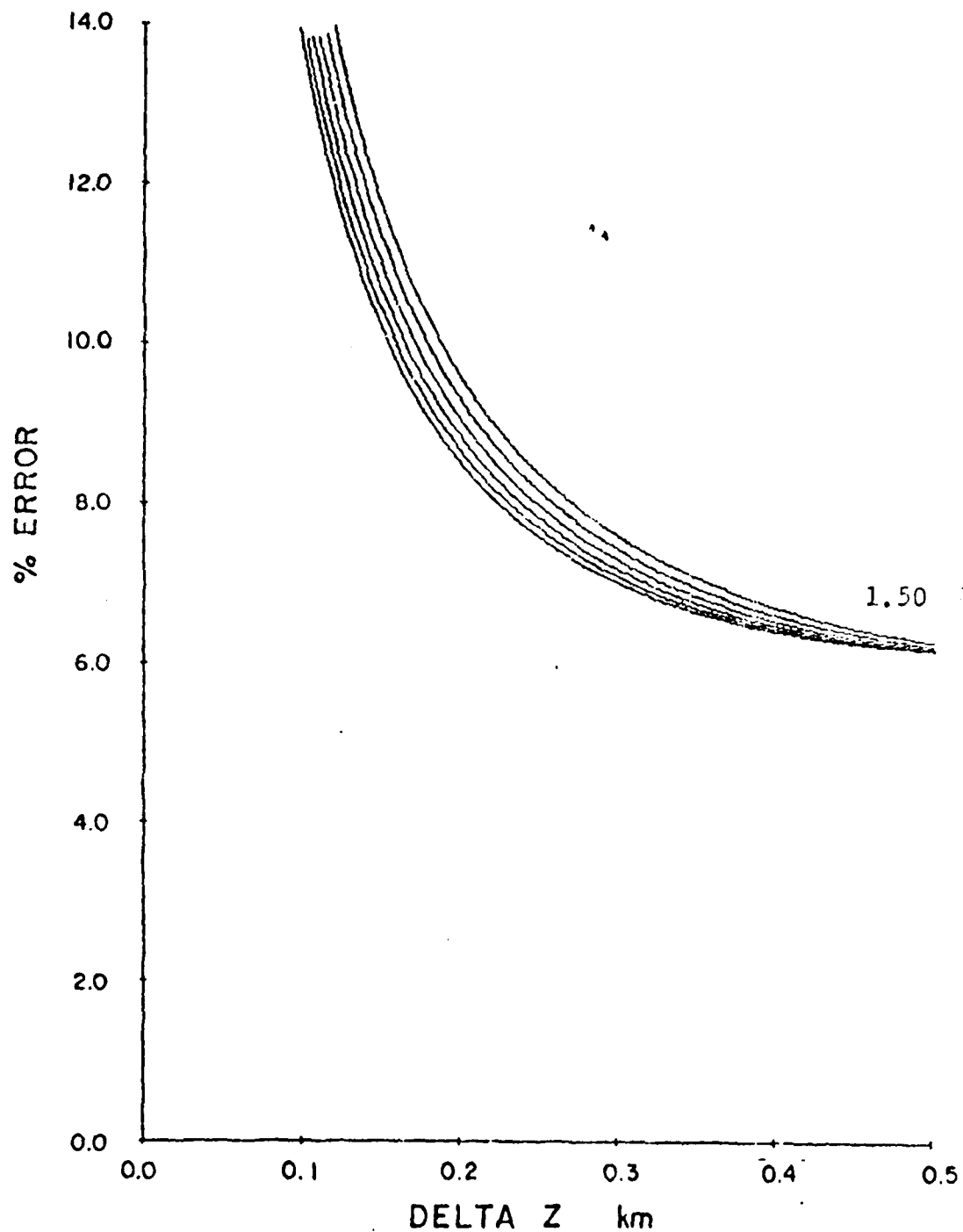


Figure 8c

BANDWIDTH 200 Hz  
ARRAY ALTITUDE 100 m  
ARRAY LENGTH 1 km

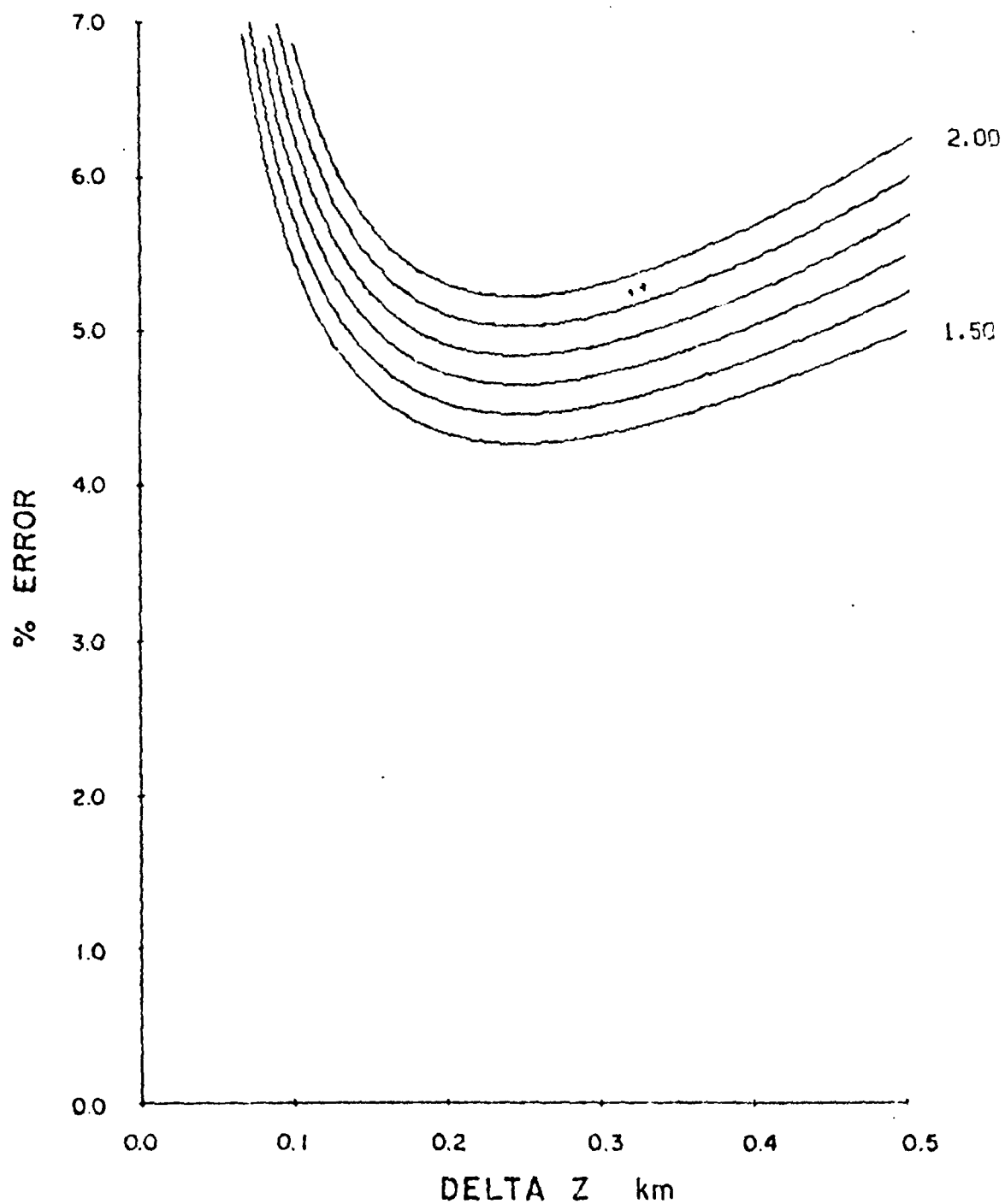


Figure 8d

BANDWIDTH 200 Hz  
ARRAY ALTITUDE 250 m  
ARRAY LENGTH 1 km

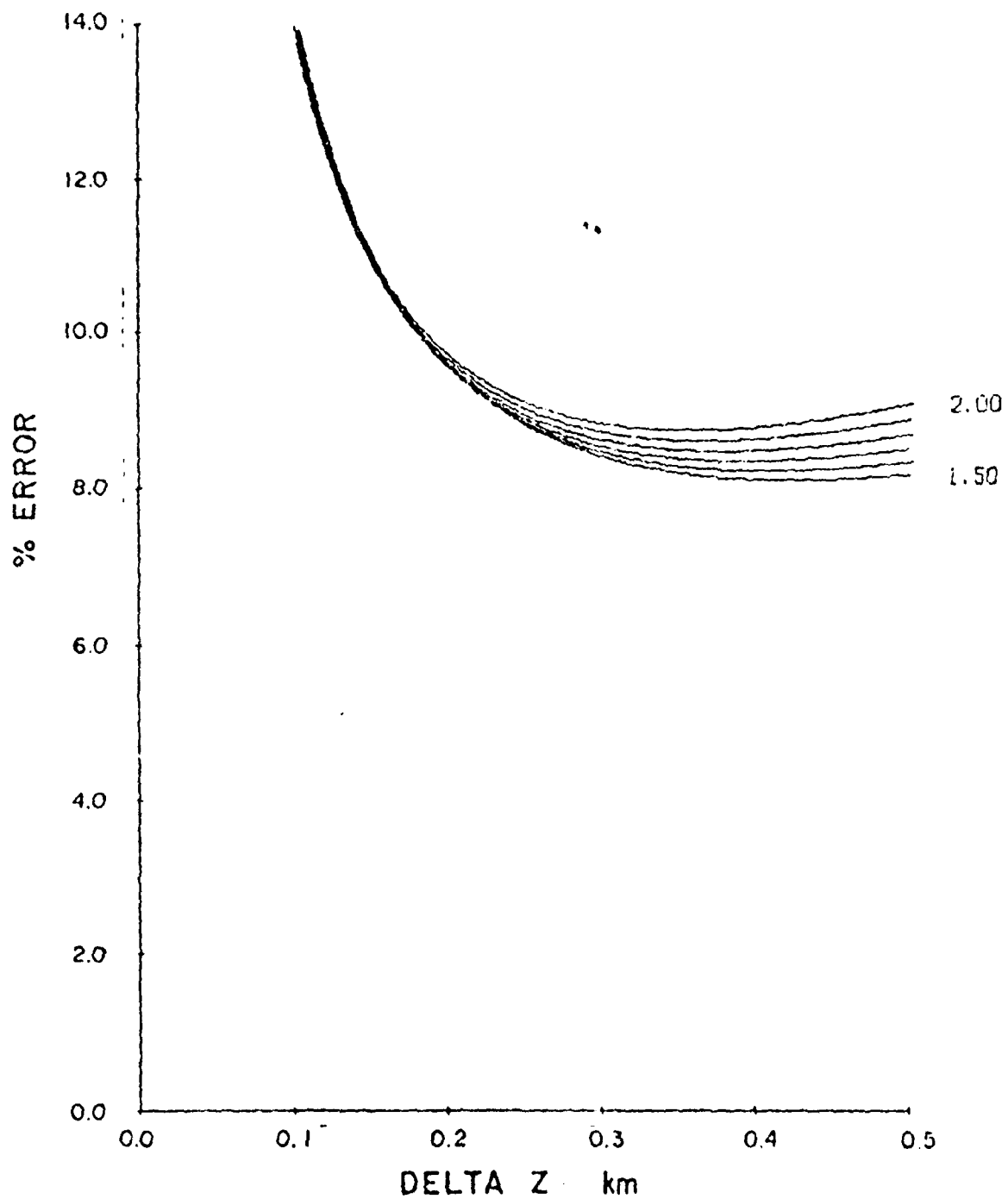


Figure 8e

BANDWIDTH 500 Hz  
ARRAY ALTITUDE 4000 m  
ARRAY LENGTH 4.8 km

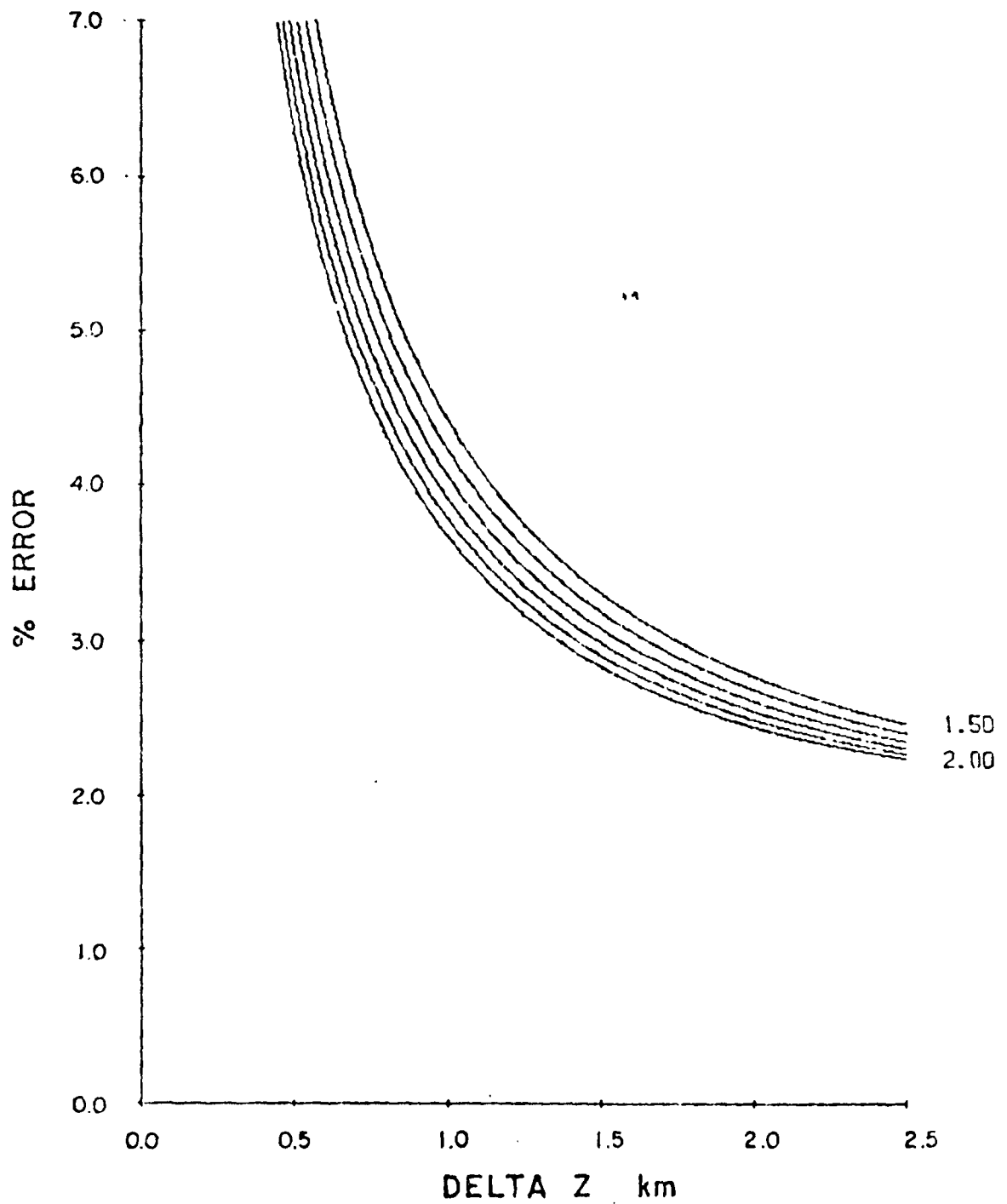


Figure 8f



BANDWIDTH 500 Hz  
ARRAY ALTITUDE 4000 m  
ARRAY LENGTH 10 km

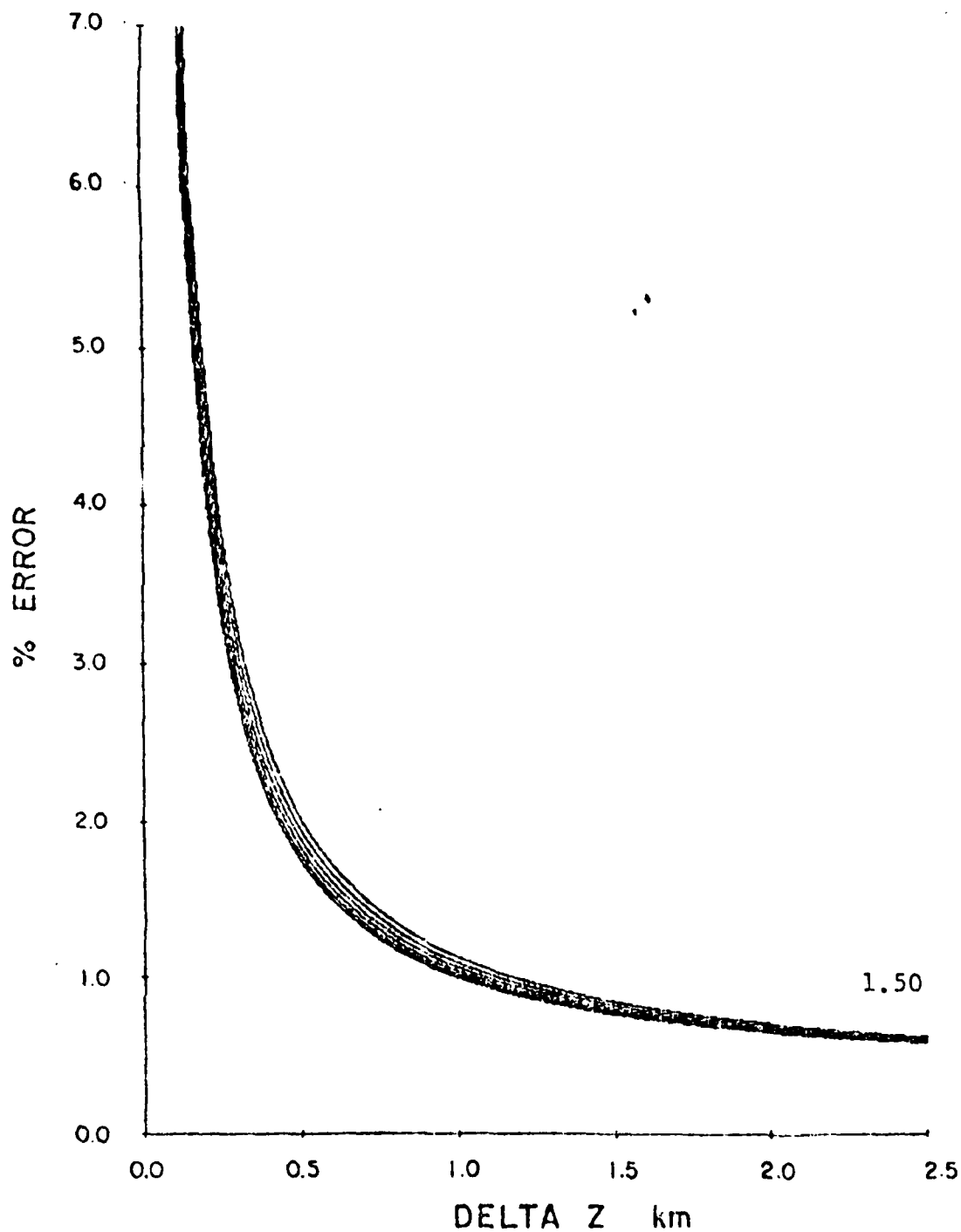


Figure 8g

# Appendix 1: Array Length Computation

```

DIMENSION X(2000), ER(2000)
C*****
C DETERMINE REQUIRED ARRAY LENGTH FOR GIVEN TARGET DEPTH,
C BANDWIDTH & RMS VELOCITY
C ENTER PARAMETERS
CALL ERASE
ACCEPT "T0 (MSECS) ? ", T0
T0=T0/1000
ACCEPT "URMS0, URMS MAX, DURMS (M/SEC) =", URMS0, URMSX, DURMS
NU=(URMSX-URMS0)/DURMS+1
DURMS=DURMS/1000
URMS0=URMS0/1000
ACCEPT "U % ERROR: MIN, MAX ? ", EMIN, EMAX
ACCEPT "SEM THRESHOLD (E.G. .5) ? ", SMIN
XMPL=4.;                                MAX ARRAY FOR PLOT
DELER=.05
EMIN=EMIN/100
EMAX=EMAX/100
DELER=DELER/100
ACCEPT "% ERROR ( IN % E.G. 1 ), X-AXIS ( IN/KM E.G. 5 ) ? ", DX, XSCAL
C*****
NE=(EMAX-EMIN)/DELER+1;                # OF ERROR STEPS
DO 1000 JPLOT=1, 4;                    LOOP FOR 4 BANDWIDTHS
CALL PLOTS(IDUM, "TAXVF")
CALL PLOT(3., 3., -3)
IF(JPLOT.EQ.1) FCUT=100
IF(JPLOT.EQ.2) FCUT=200
IF(JPLOT.EQ.3) FCUT=500
IF(JPLOT.EQ.4) FCUT=1000
DT=1./FCUT;                            TIME DURATION OF PULSE
C*****
C DRAW AXES
CALL AXIS(0., 0., 71% ERROR, -7, DX*100*NE*DELER, 0., 0., 1./DX)
XL=XSCAL*XMPL
IF(XL.GT.10) XL=XSCAL
CALL AXIS(0., 0., SHARPAY, 5, XL, 90., 0., 1./XSCAL)
CALL PLOT(0., 0., 3)
C*****
C LOOP FOR ALL RMS VELOCITIES
DO 200 JJ=1, NU
URMS=URMS0+(JJ-1)*DURMS;                NEW URMS
JER=1
DO 50 J=1, NE;                          LOOP FOR ALL ERRORS
EU=EMIN+(J-1)*DELER;                    GET ERROR THIS CASE
TT=T0*T0/(DT*DT)
TT=EU*EU*TT
RAD1=TT+0.25
RAD1=SQRT(RAD1)
RAD2=RAD1+0.5
RAD2=SQRT(RAD2)
X(JER)=URMS*DT*RAD2/EU;                X FROM EQ, NO SEM CONDITION YET
X(JER)=X(JER)/SMIN;                    USE SEM MIN TO GET REQUIRED ARRAY SIZE
ER(JER)=EU*100;                        % ERROR
IF(X(JER).GT.XMPL) GO TO 50
IF(JER.EQ.1) EG=EU;                    SAVE 1ST ERROR ON PLOT
JEND=JER
JER=JER+1
50 CONTINUE
C*****
C PLOT RESULTS THIS CASE
IPEN=3
DO 100 K=1, JEND

```

```

      XX=100*E0*DX+100*(K-1)*DX*DELER
      YY=X(K)*XSCAL
      CALL PLOT(XX,YY,IFEN)
100   IFEN=2
      CALL NUMBER(XX+.25,YY,.1,URMS,0.,3)
      CALL PLOT(0.,0.,3)
200   CONTINUE
C*****
C CLOSE PLOT FILE, PLOT
      CALL PLOT(0.,-2.,3)
      CALL SYMBOL(999.,999.,.2,4H TO ,0.,4)
      CALL NUMBER(999.,999.,.2,10,0.,2)
      CALL SYMBOL(999.,999.,.2,14H BANDWIDTH ,0.,14)
      CALL NUMBER(999.,999.,.2,FCUT,0.,1)
      CALL PLOT(0.,0.,999)
      CALL FSWAP("AGPLT SU" )
      DO 500 JJJ=1,1
      CALL FSWAP("FPBITS.SU")
      CALL FPUOK
500   CONTINUE
1000  CONTINUE
      CALL RESET
      CALL EXIT
      END

```

## Appendix II: Interval Velocity Error Computation

```

C*****
C DETERMINE INTERVAL VELOCITY ERROR (%)
C FROM ARRAY VELOCITY & TWO WAY TRAVEL TIME MEASUREMENTS
C ENTER PARAMETERS:
  CALL ERASE
  ACCEPT"BANDWIDTH (HZ) ? ",BAND
  ACCEPT"ARRAY LENGTH ? ",XARRAY
  ACCEPT"ARRAY ALTITUDE (KM) ? ",AH
  ACCEPT"INTERVAL: UMIN, UMAX, DELT (E.G. 1.5,2.,.1) ? ",UMIN,UMAX,DELT
  ACCEPT"O-TIME, 1-DEPTH ? ",IDTP
  IF(IDTP.EQ.0) ACCEPT"MAXIMUM LAYER THICKNESS (SEC) ? ",TH
  IF(IDTP.EQ.1) ACCEPT"MAXIMUM LAYER THICKNESS (KM) ? ",TH
  IF(IDTP.EQ.0) ACCEPT"IN/SEC (E.G. 5) ? ",DX
  IF(IDTP.EQ.1) ACCEPT"IN/KM (E.G. 5) ? ",DX
  ACCEPT"MAX % ERROR, IN/ %ERROR ? ",EMAX,DY
  N=TH*BAND;          # OF STEPS
  DELT=1./BAND;       DELTA T
  NU=(UMAX-UMIN)/DELT; # OF VELOCITIES
C*****
C OPEN PLOT FILES. DRAW AXES
  CALL PLOTS(IDUN,"TAXUF")
  CALL PLOT(3.,10.,-3)
  IF(IDTP.EQ.0)CALL AXIS(0.,0.,14HDELTA TO (SEC),-14,TH*DX,0.,0.,1./DX
  IF(IDTP.EQ.1) CALL AXIS(0.,0.,12HDELTA Z (KM),-12,TH*DX,0.,0.,1./DX
  CALL AXIS(0.,0.,7H% ERROR,7,EMAX*DY,90.,0.,1./DY)
C*****
C 1ST LAYER IS FIXED, GET URMS (U1) & T0 (T1), URMS ERROR (UER1)
C & U1 LOW (U1L), & U1 HIGH (U1H)
C DEFINE AFFECTIVE ARRAY LENGTH**2 (XAR2)
  U1=1.5
  T1=2*AH/U1
  XAR2=(.5*XARRAY)**2;          (ASSUMSC SEM =.5)
  U12=U1**2
  UER1=DELT*U12*SQR(T1**2+XAR2/U12)/XAR2
  U1L=U12*(1.-UER1)**2*(T1+DELT)
  U1H=U12*(1.+UER1)**2*(T1-DELT)
C*****
C LOOP FOR ALL VELOCITIES
  DO 100 J=1,NU
    UINT=UMIN+(J-1)*DELT;          NEW INTERVAL VELOCITY
    UINT2=UINT**2
C*****
    IFEN=3
    IF(IDTP.EQ.1) N=BAND*2*TH/UINT+1
C*****
C LOOP OVER ALL THICKNESSES
  50 DO 90 J=4,N
    DT=J*DELT,          NEXT LAYER THICKNESS
    T2=T1+DT
    URMS2=(UINT2*DT+U12*T1)/T2;          2ND URMS (TRUE)
    UER2=DELT*URMS2*SQR(T2**2+XAR2/URMS2)/XAR2; % ERROR THIS URMS
    U2H=URMS2*(1.+UER2)**2*(T2-DELT);          U2 RMS HIGH**2 * T0 - DELTA
    UMAX=(U2H-U1L)/(T2-T1-2*DELT);          UINT MAX **2
    U2L=URMS2*(1.-UER2)**2*(T2+DELT);          U2 RMS LOW**2 * T0 + DELTA
    UMIN=(U2L-U1H)/(T2-T1+2*DELT);          UINT MIN **2
    IF(UMAX.LT.0) GO TO 90
    IF(UMIN.LT.0) GO TO 90
    UMAX=SQRT(UMAX)
    UMIN=SQRT(UMIN)
    DU=UMAX-UMIN,          DELTA UINT
    DUINT=.5*DU/UINT;          % UINT ERROR
    IF(DUINT.LT.0) GO TO 90
  90

```

```

      IF(DVINT.GT..01*EMAX) GO TO 90
C*****
C PLOT % (INT ERROR
      XX=DT*DX
      IF(IDTP.EQ.1) XX=DX*DT*VINT/2
      YY=100*DX*INT*DY
      CALL PLOT(XX,YY,IPEN)
      IPEN=2
90    CONTINUE
      IF(JV.EQ.1.OR.JV.EQ.NV) CALL NUMBER(XX+.25,YY-.05,.1,VINT,0.,2)
100   CONTINUE
C*****
C ANNOTATION, CLOSE PLOTFILE, PLOT
      CALL PLOT(TH*DX/2,EMAX*DY,-3)
      CALL SYMBOL(0.,0.,.1,15BANDWIDTH(HZ),0.,15)
      CALL NUMBER(999.,999.,.1,BAND,0.,1)
      CALL PLOT(0.,-2,-3)
      CALL SYMBOL(0.,0.,.1,15HARRAY ALTITUDE (M),0.,19)
      CALL NUMBER(999.,999.,.1,AH*1000,0.,1)
      CALL PLOT(0.,-2,-3)
      CALL SYMBOL(0.,0.,.1,15HARRAY LENGTH (KM),0.,18)
      CALL NUMBER(999.,999.,.1,XARRAY,0.,1)
      CALL PLOT(0.,0.,999)
      CALL FSWAP("AGPLT.SU")
      CALL FSWAP("FPBITS.SU")
      CALL EXIT
      END

```

